Models Lecture #5

The distance ladder

PHILOSOPHIÆ NATURALIS PRINCIPIA MATHEMATICA.

Autore *JS. NEWTON*, *Trin. Coll. Cantab. Soc.* Matheseos Professore *Lucasiano*, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, *Reg. Soc.* PRÆSES.
Julii 5. 1686.

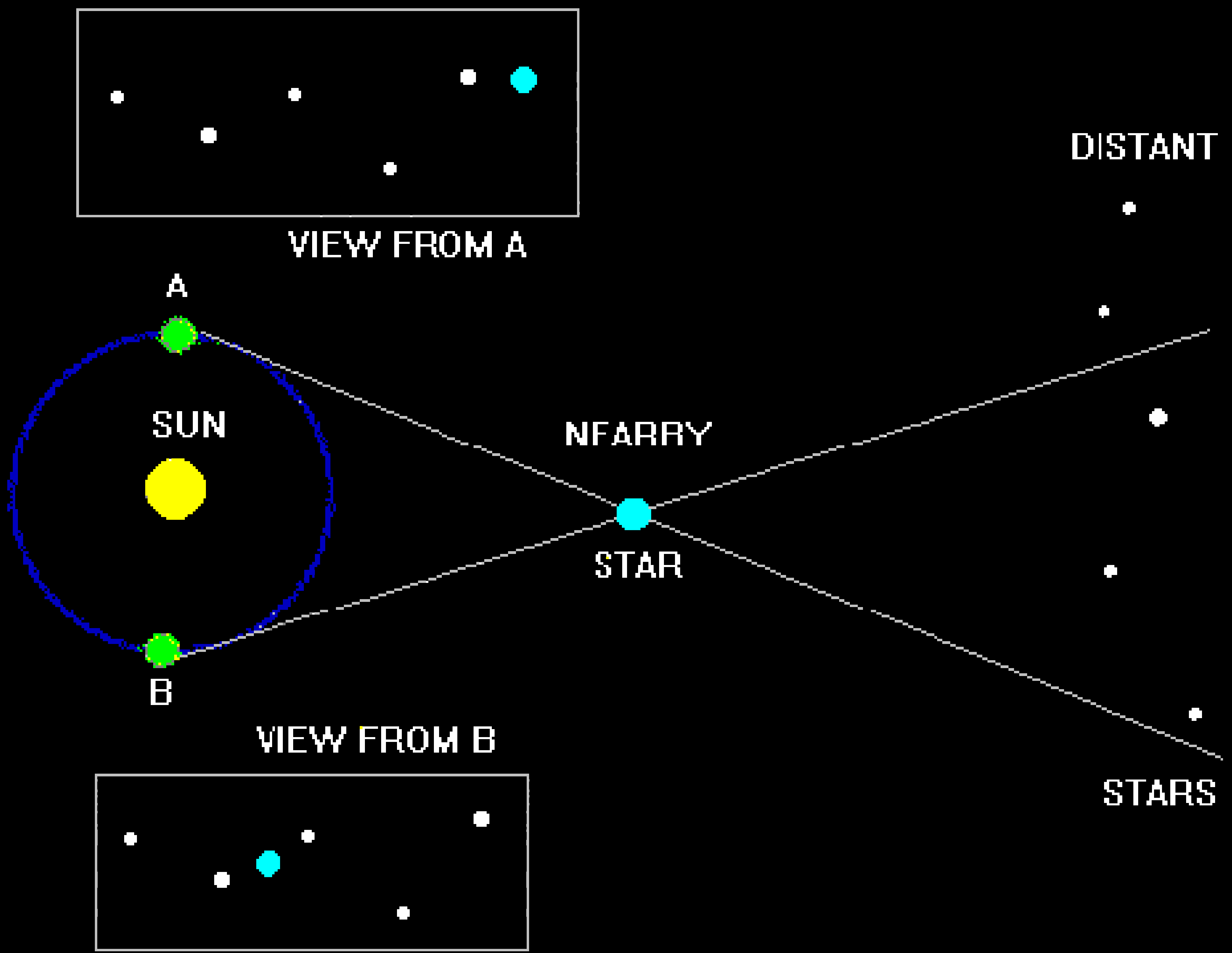
LONDINI,

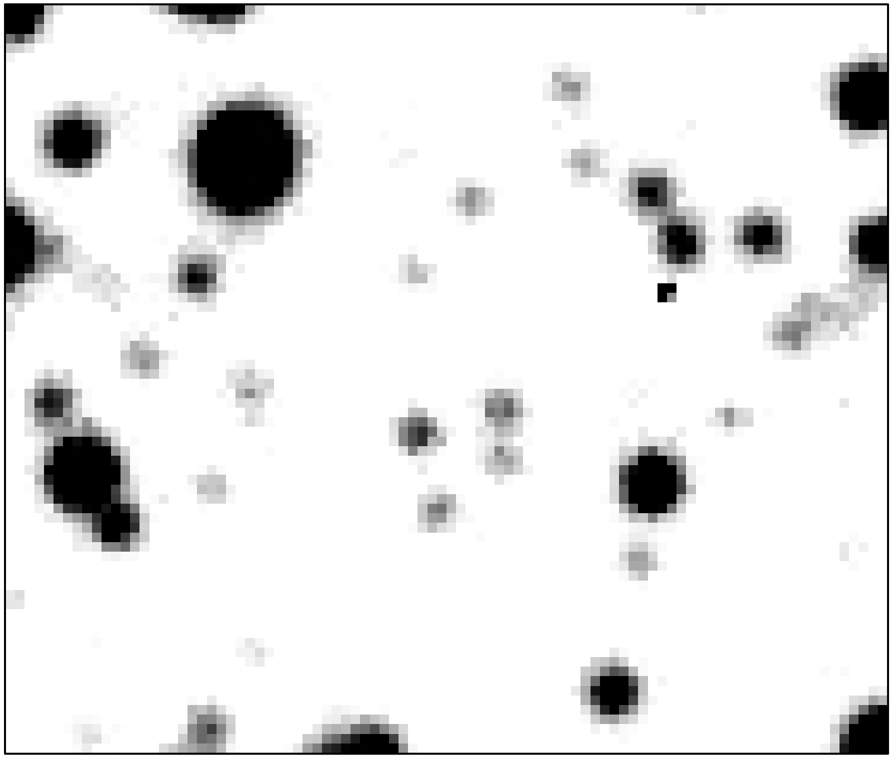
Jussu *Societatis Regiæ* ac Typis *Josephi Streater*. Prostat apud plures Bibliopolas. *Anno* MDCLXXXVII.

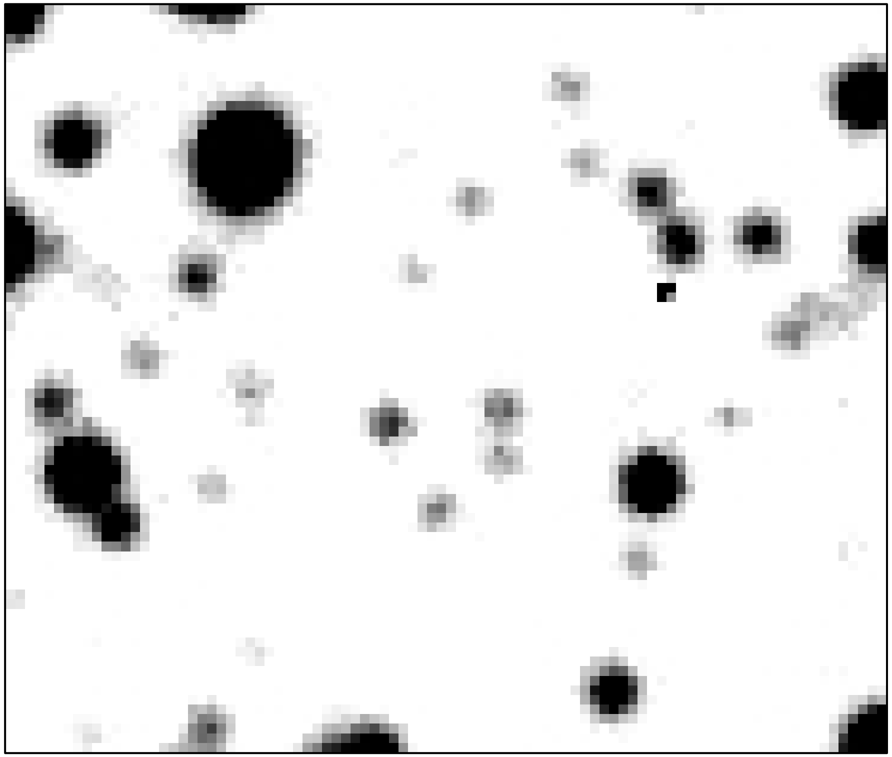
Pieter Bruegel the Elder, ca. 1563

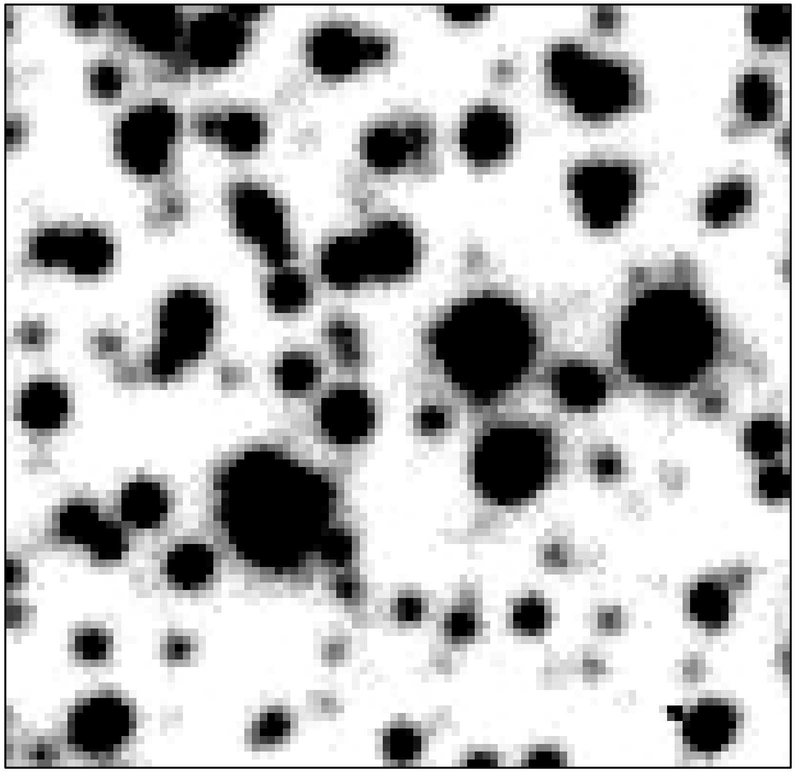
The Cosmological Distance Scale

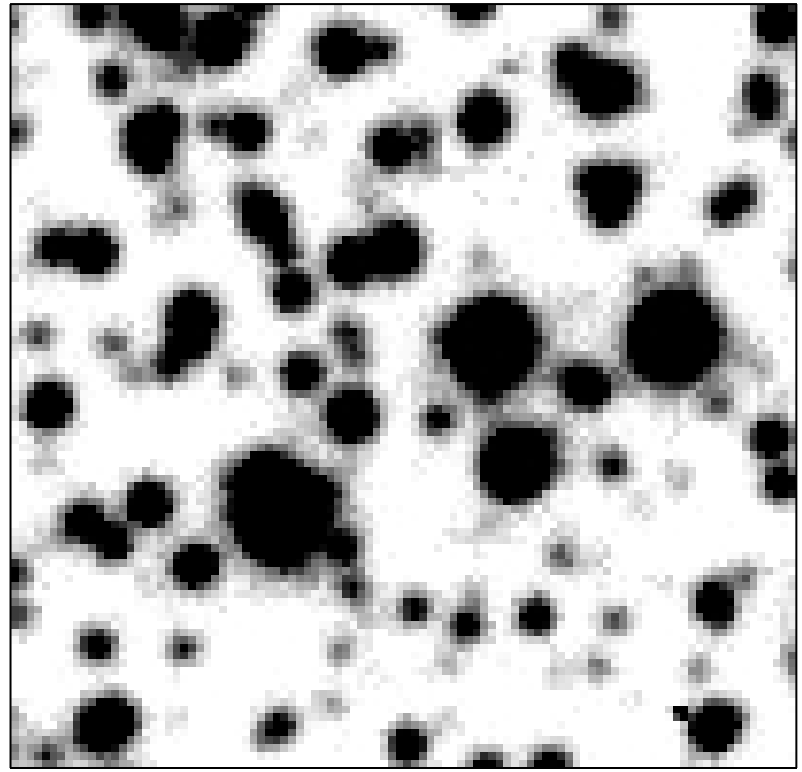

1 2 3 4 5 6 7

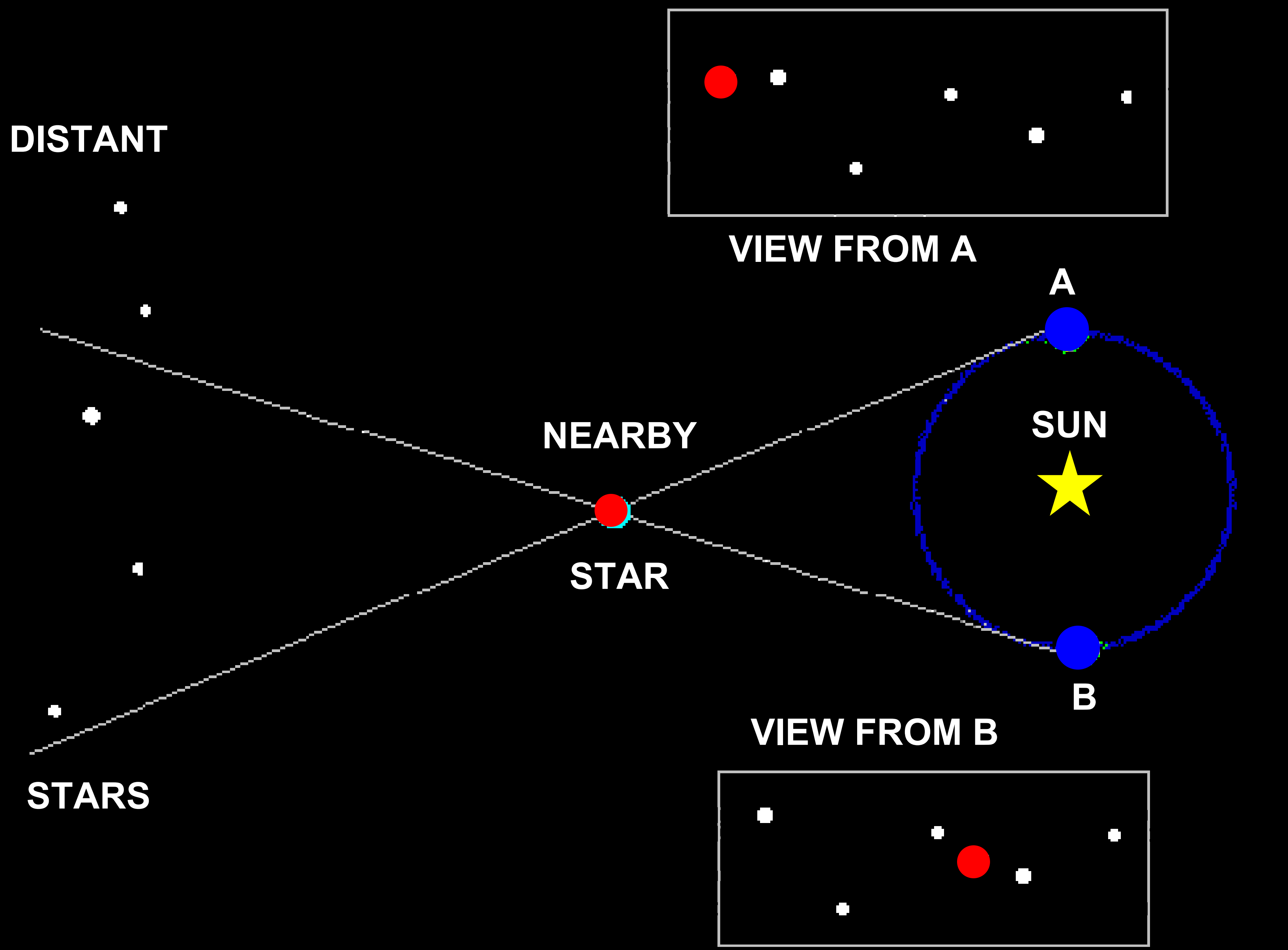

VIEW FROM A
A
SUN
NFARRY
STAR
DISTANT
STARS
B
VIEW FROM B



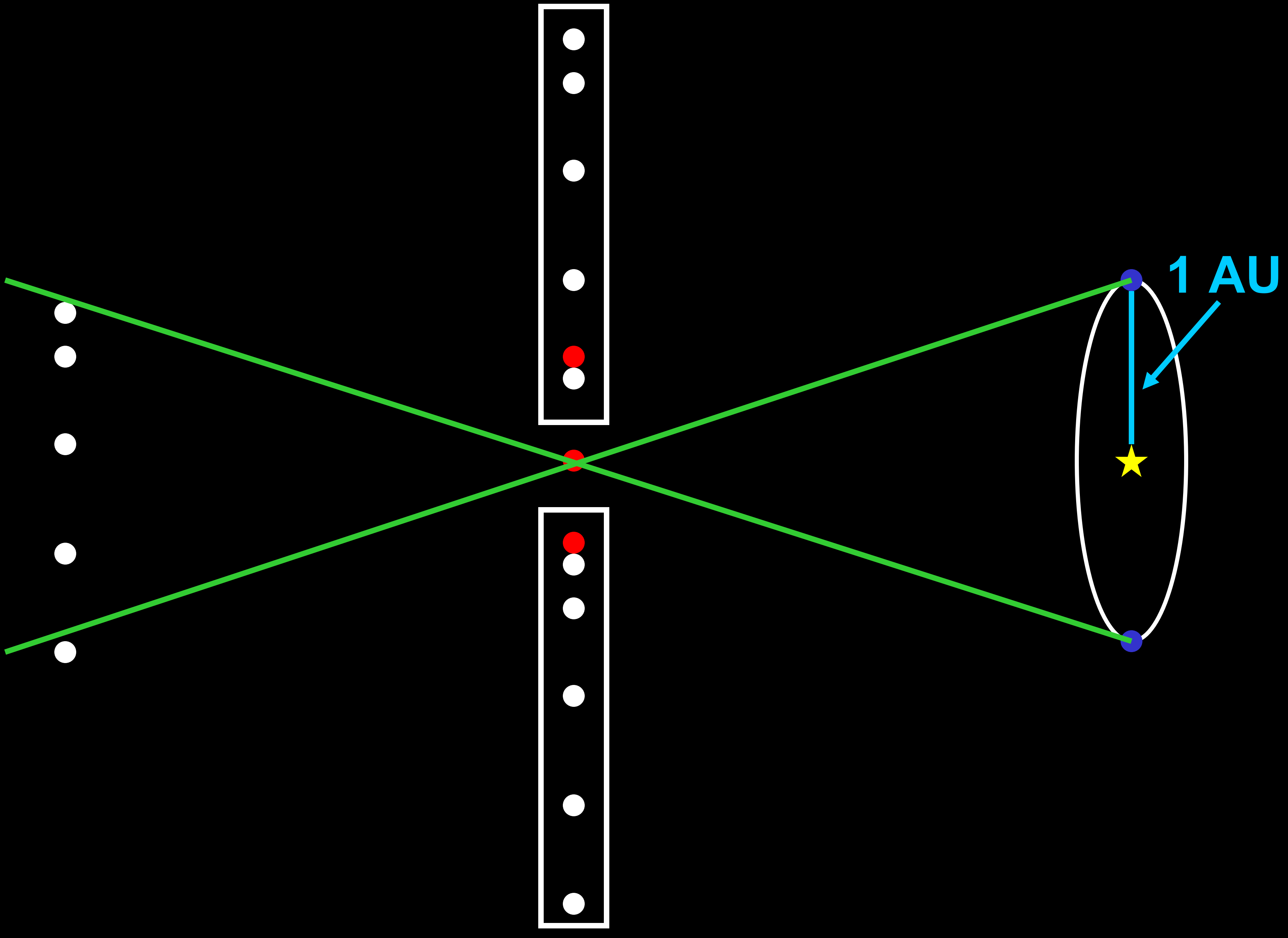


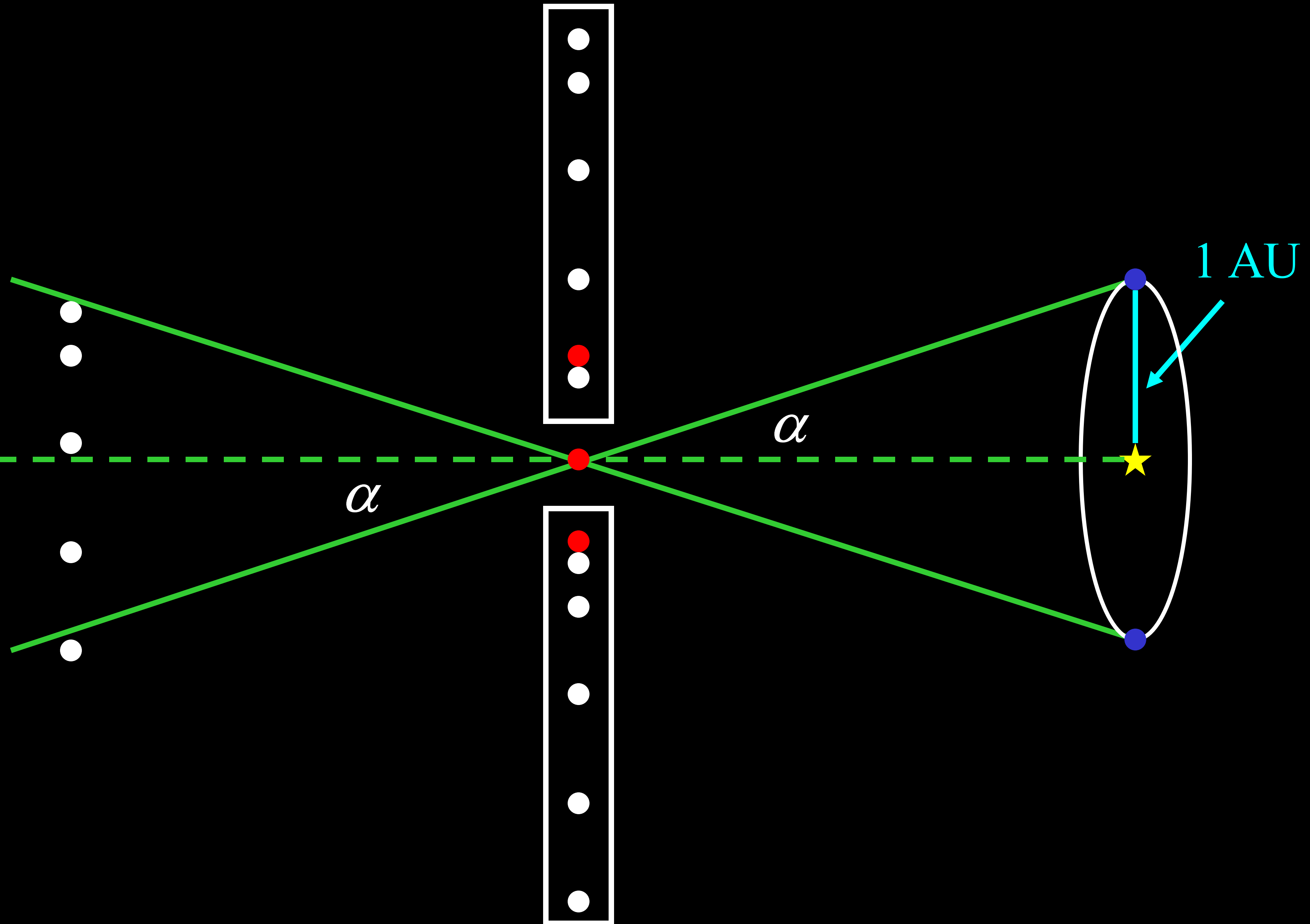




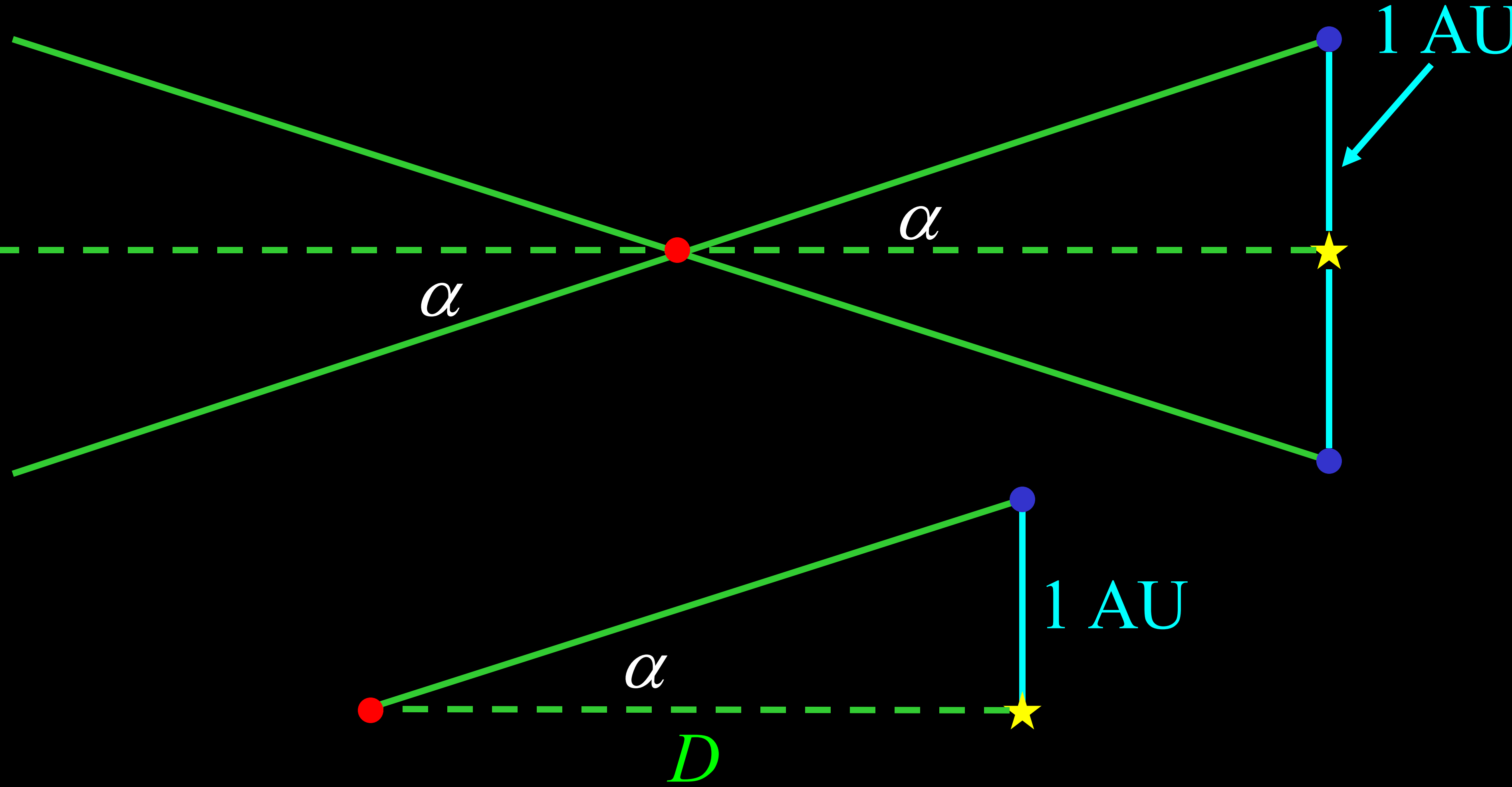


DISTANT
STARS
NEARBY
STAR
SUN
A
B
VIEW FROM A
VIEW FROM B

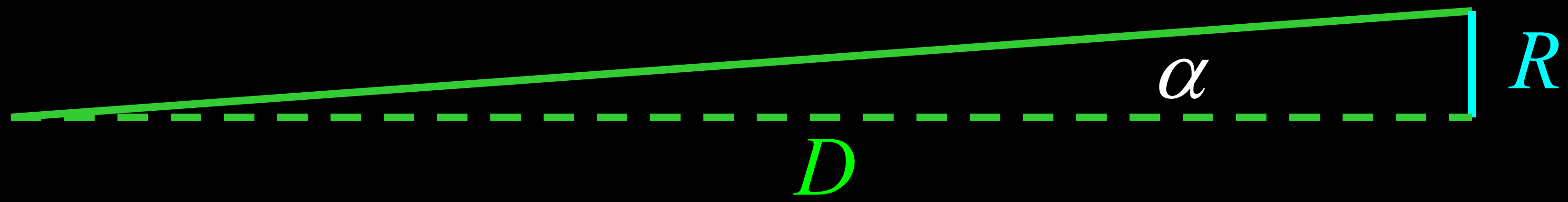

1 AU



1 AU
α
α



$$\tan\alpha = \frac{1\ \mathrm{AU}}{D}$$



$$\tan\alpha = \frac{R}{D}$$

law of skinny triangles:

$$\tan\alpha = \sin\alpha = \alpha \quad \text{(in radians)}$$

$$\alpha \quad \text{(in radians)} = \frac{R}{D}$$

What's a radian?

$$2\pi \text{ radians} = 360 \text{ degrees}$$

$$1 \text{ radian} = \frac{360}{2\pi} \text{ degrees} \sim 60 \text{ degrees}$$

F 70

B C 60

P T E O 50

B Z F E D 40

O F C L T B 30

T E P O L F D Z 20

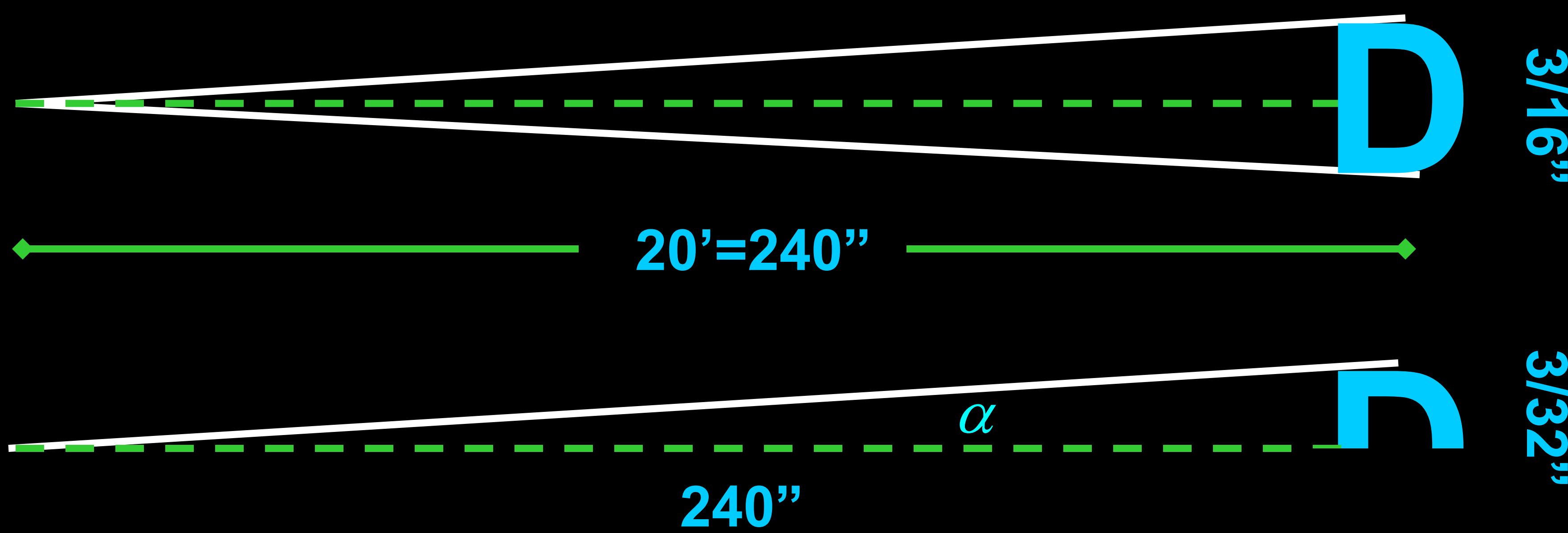
L P C T Z D B F E O 15

Z O E C F L D P B T 10

E T O L E B Z E F D C 7

B O F C P T E B L F B E Z C O P E 4

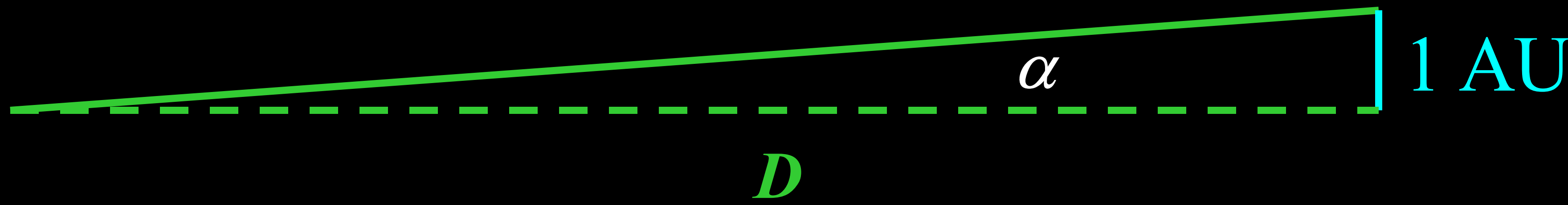
How good are your eyes?



$$\alpha = \frac{3}{32}\,\frac{1}{240} = 4\times10^{-4}\ \cancel{\text{radians}} \times \frac{360\ \text{degrees}}{2\pi\ \cancel{\text{radians}}} = 0.02^\circ$$

$$\alpha = 0.02^\circ \times \frac{60\ \text{minutes}}{1\ \text{degree}} = 1'$$

D $= 2'$

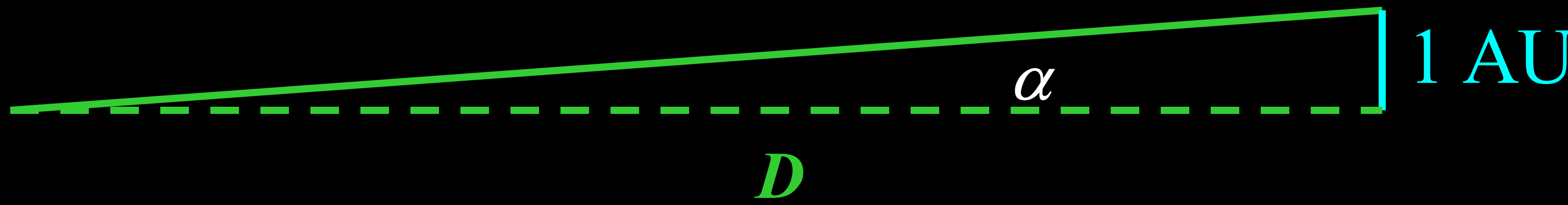


$$\alpha = \frac{1\ \text{AU}}{D}\ \cancel{\text{radians}} \times \frac{60\ \text{degrees}}{\cancel{\text{radian}}}$$

$$\alpha = \frac{60\ \text{AU}}{D}\ \cancel{\text{degrees}} \times \frac{60\ \text{minutes}}{1\ \cancel{\text{degree}}}$$

$$\alpha = \frac{3600\ \text{AU}}{D}\ \cancel{\text{minutes}} \times \frac{60\ \text{seconds}}{1\ \cancel{\text{minute}}}$$

$$\alpha = \frac{206{,}264.8\ \text{AU}}{D}\ \text{seconds}$$



$$\alpha = \frac{1\ \text{AU}}{D}\ \text{radians}$$

$$\alpha = \frac{206{,}264.8\ \text{AU}}{D}\ \text{seconds}$$

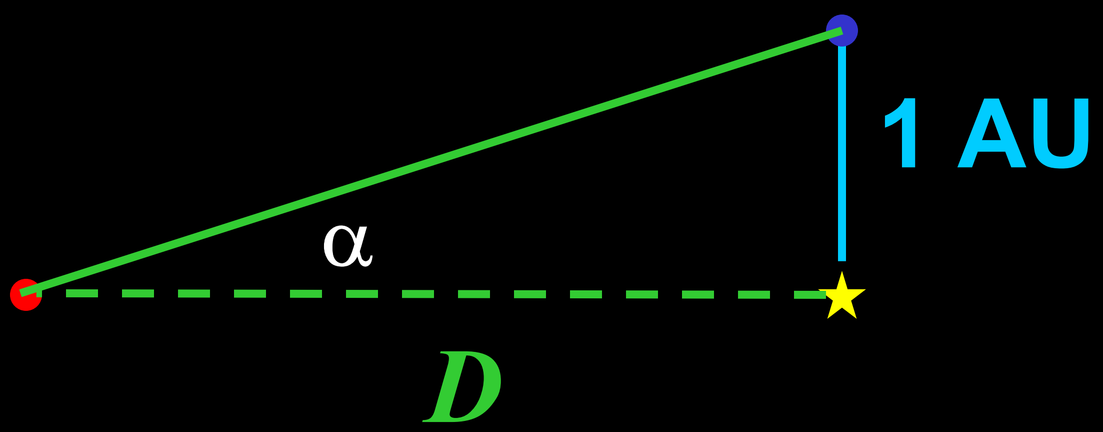
1 pc = 206,264.8 AU = 3.26 light years

= 10^{13} (10,000,000,000,000) miles

$$\alpha = \frac{\text{pc}}{D}\ \text{seconds} \qquad D = \frac{\text{second}}{\alpha}\ \text{pc}$$

$$\frac{\text{D}}{200{,}000\ \text{AU}} = \frac{\text{seconds}}{\alpha}$$

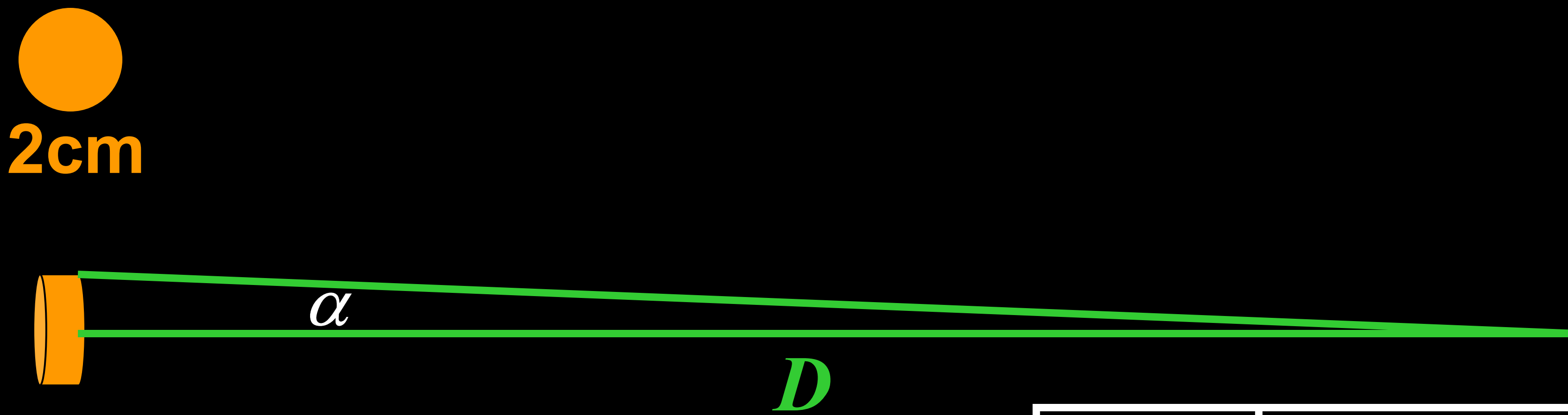
$$\frac{\text{D}}{\text{pc}} = \frac{\text{seconds}}{\alpha}$$



$$\frac{D}{\text{pc}} = \frac{\text{seconds}}{\text{parallax}}$$

star	parallax (”)	distance (pc)
α Centauri	0.75	1.3
Barnard’s star	0.5	2.0
Sirius	0.4	2.5
Altair	0.2	5.0

Let's think for a second of arc



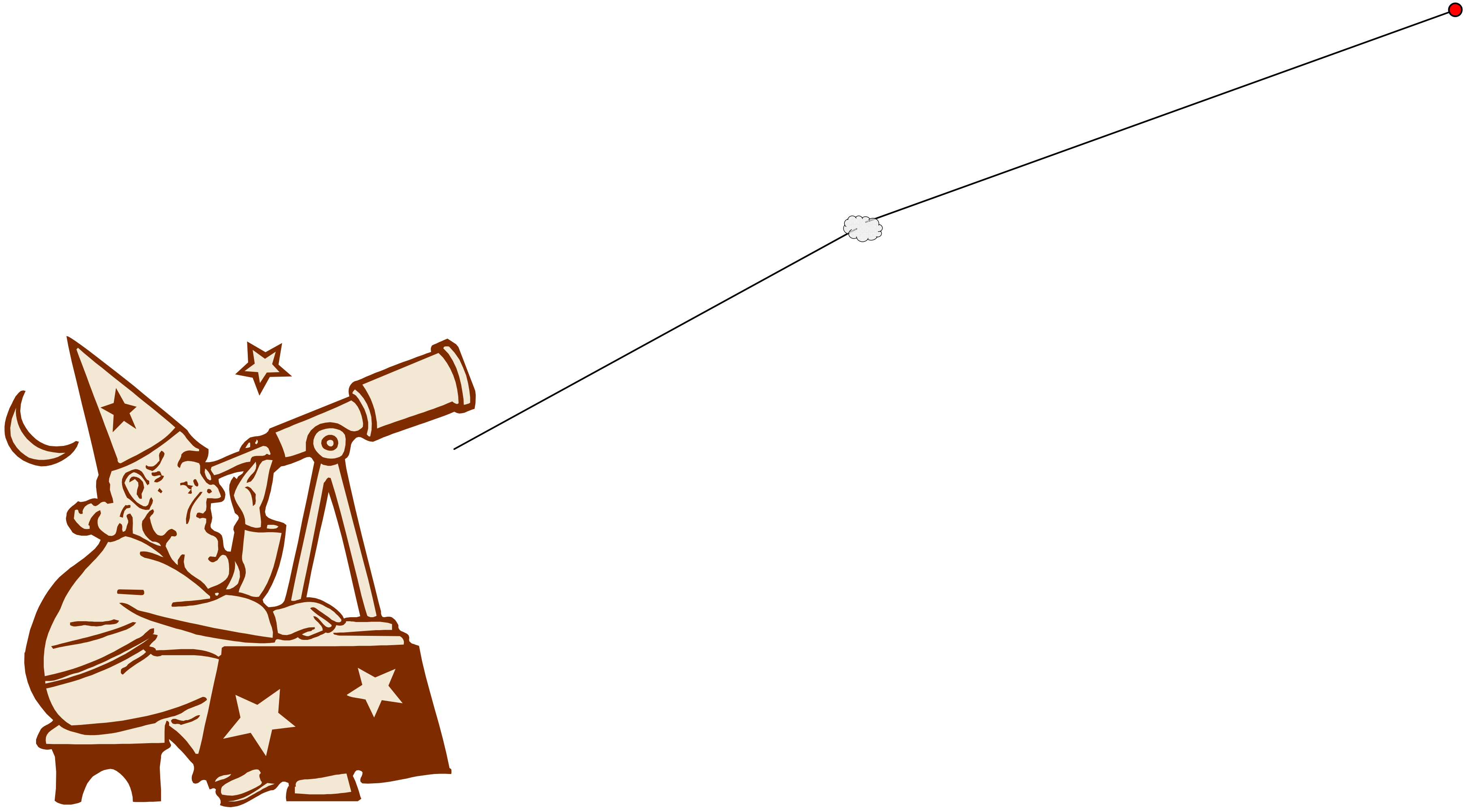
$$\alpha = \frac{1\ \text{cm}}{D}\ \text{radians}$$

$$\alpha = \frac{200{,}000\ \text{cm}}{D}\ \text{seconds}$$

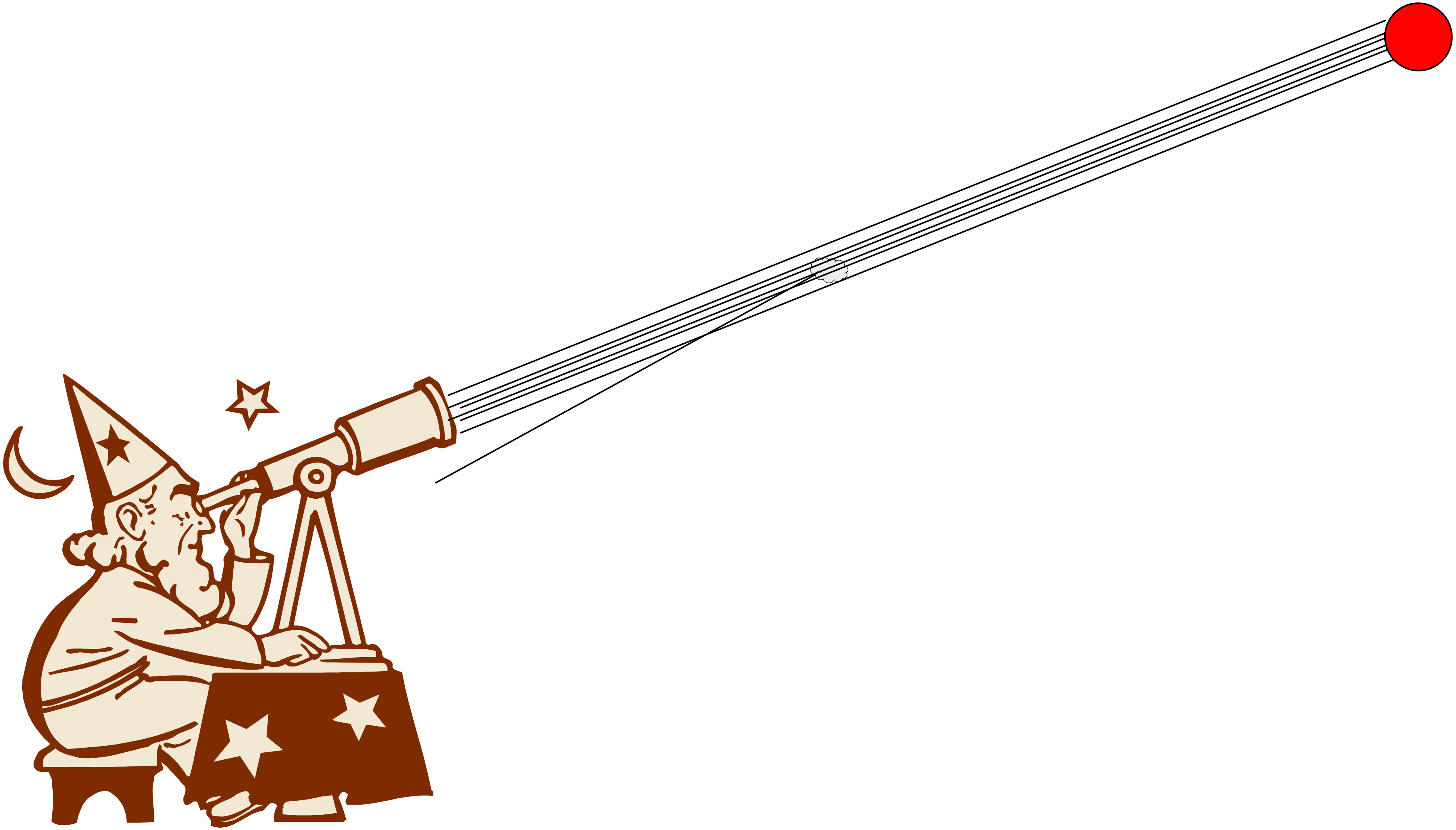
$$\alpha = \frac{2\ \text{km}}{D}\ \text{seconds}$$

α	D
4"	½ km
2"	1 km
1"	2 km
0.1"	20 km
0.01"	200 km
0"	infinity

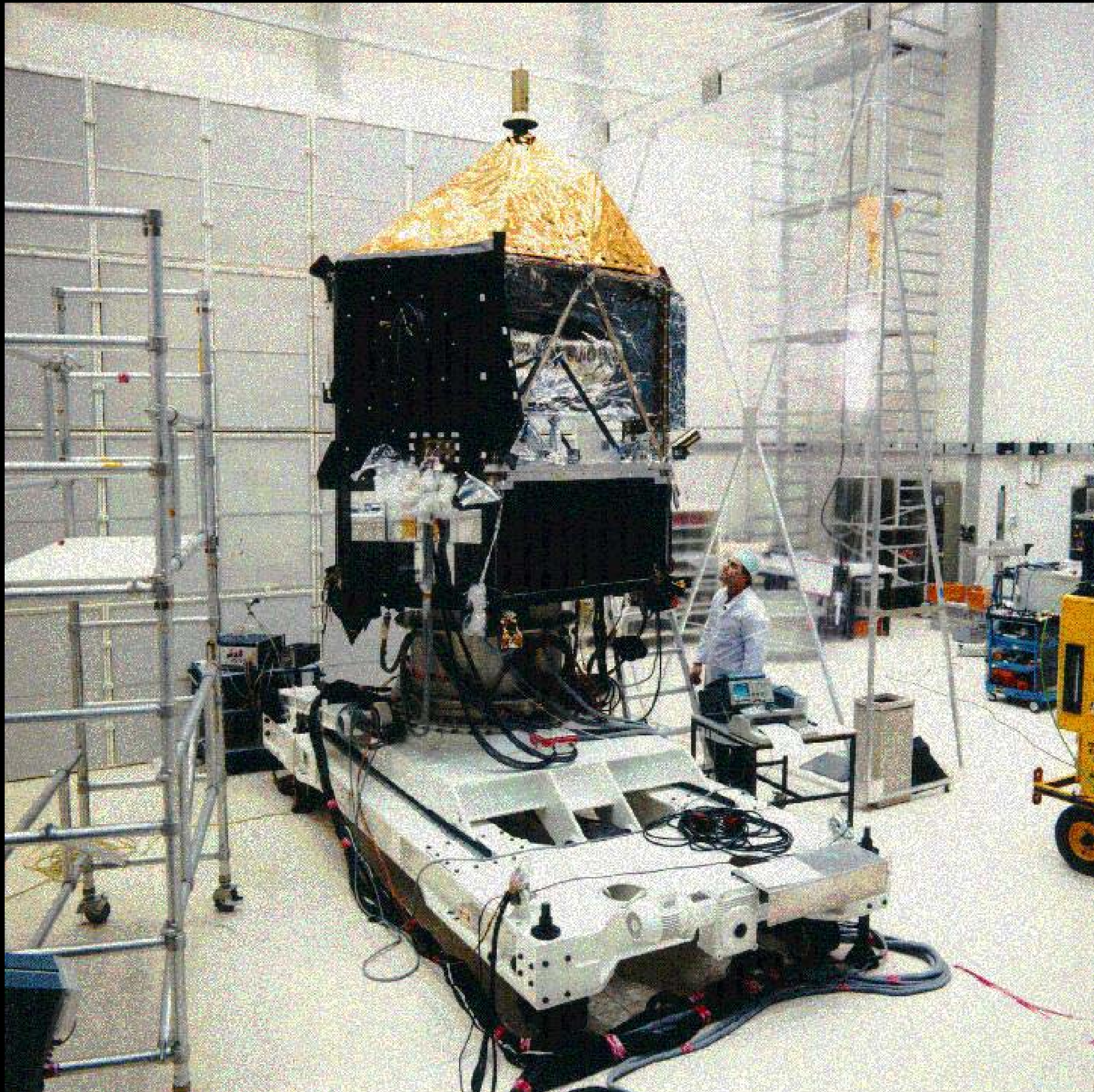
Twinkle, twinkle little star

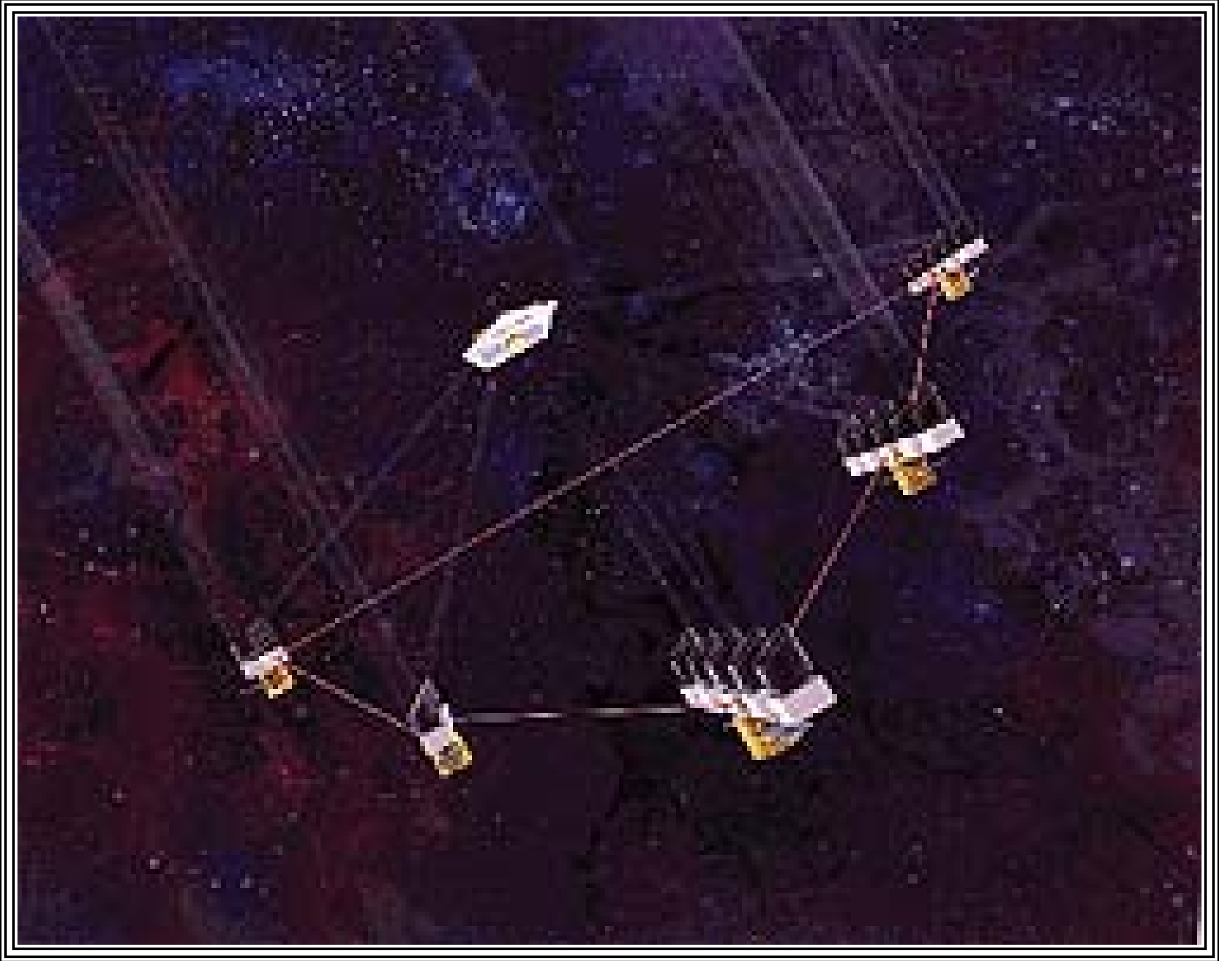


Twinkle, twinkle little star



Hipparcos



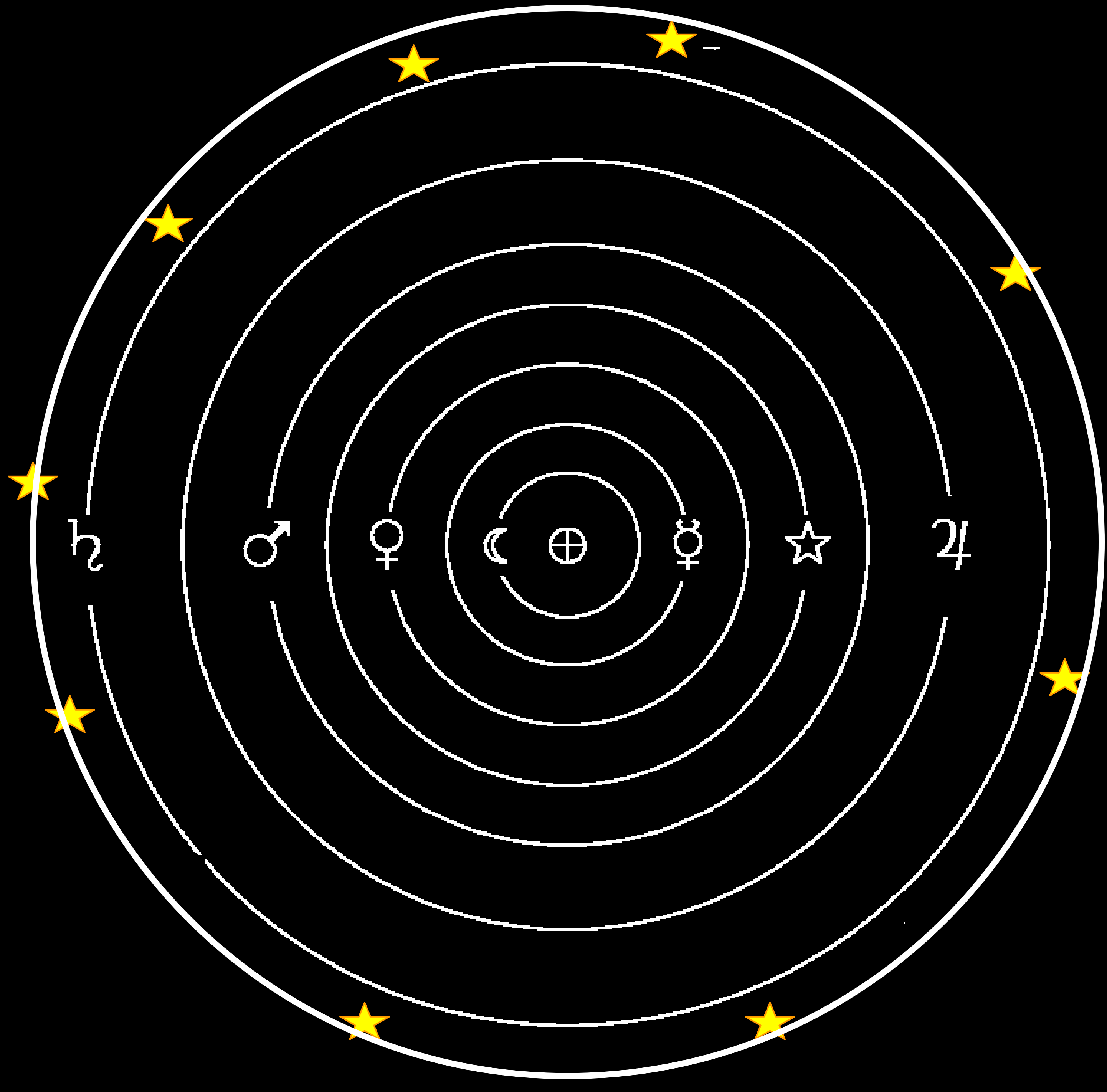


Planet Imager

Formation Flying

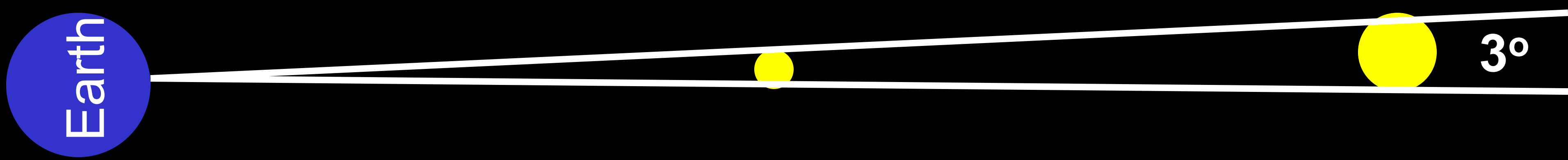
Launch: 2030

32 X 8 meter mirrors
Baseline = 6000 km

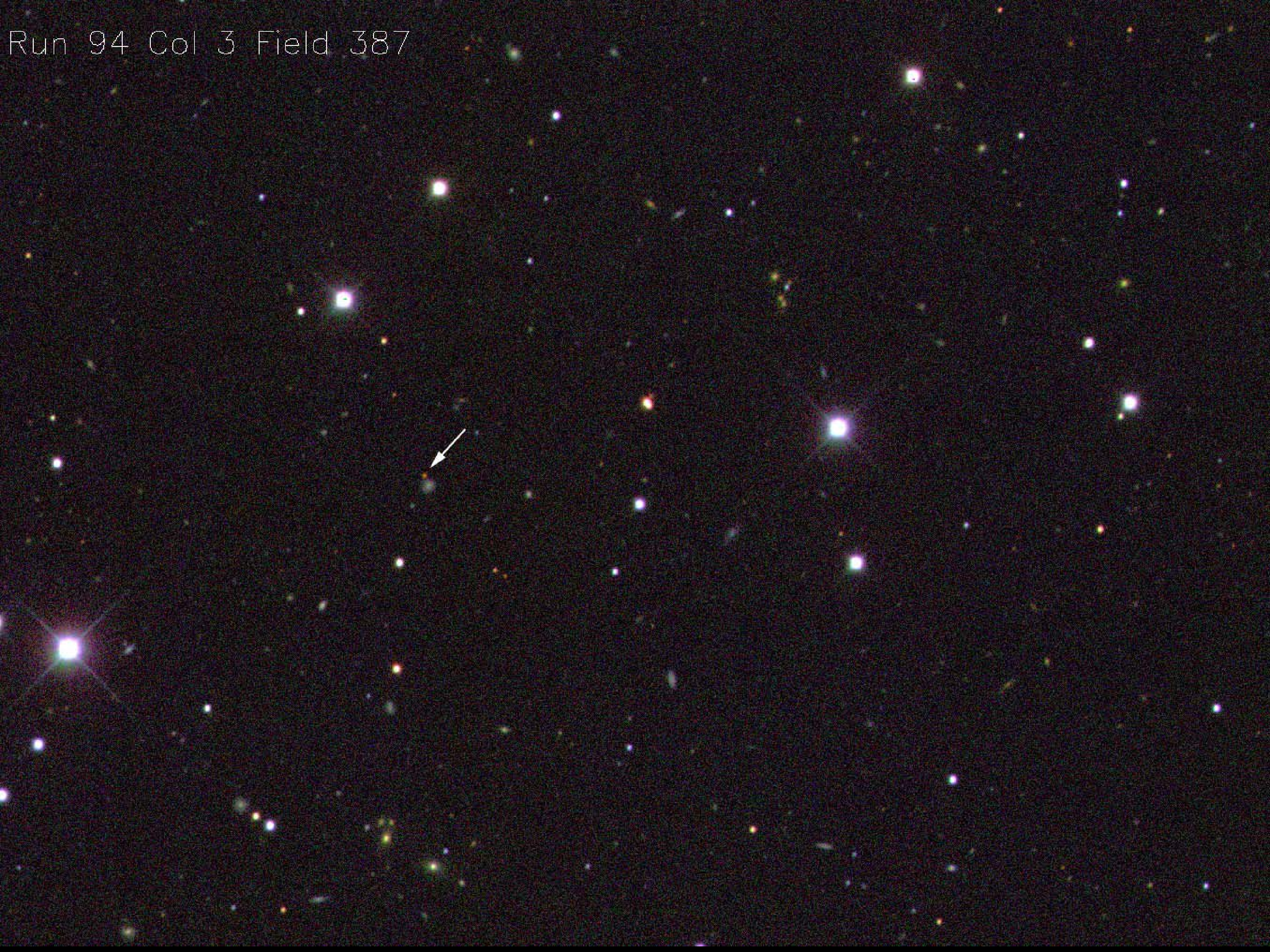


Planet	angular diameter (in minutes) Ptolemy	True
Mercury	2	0.01
Venus	3	0.5
Mars	1.5	0.15
Jupiter	2.5	0.4
Saturn	1.7	0.2
Bright stars	1.5	~0

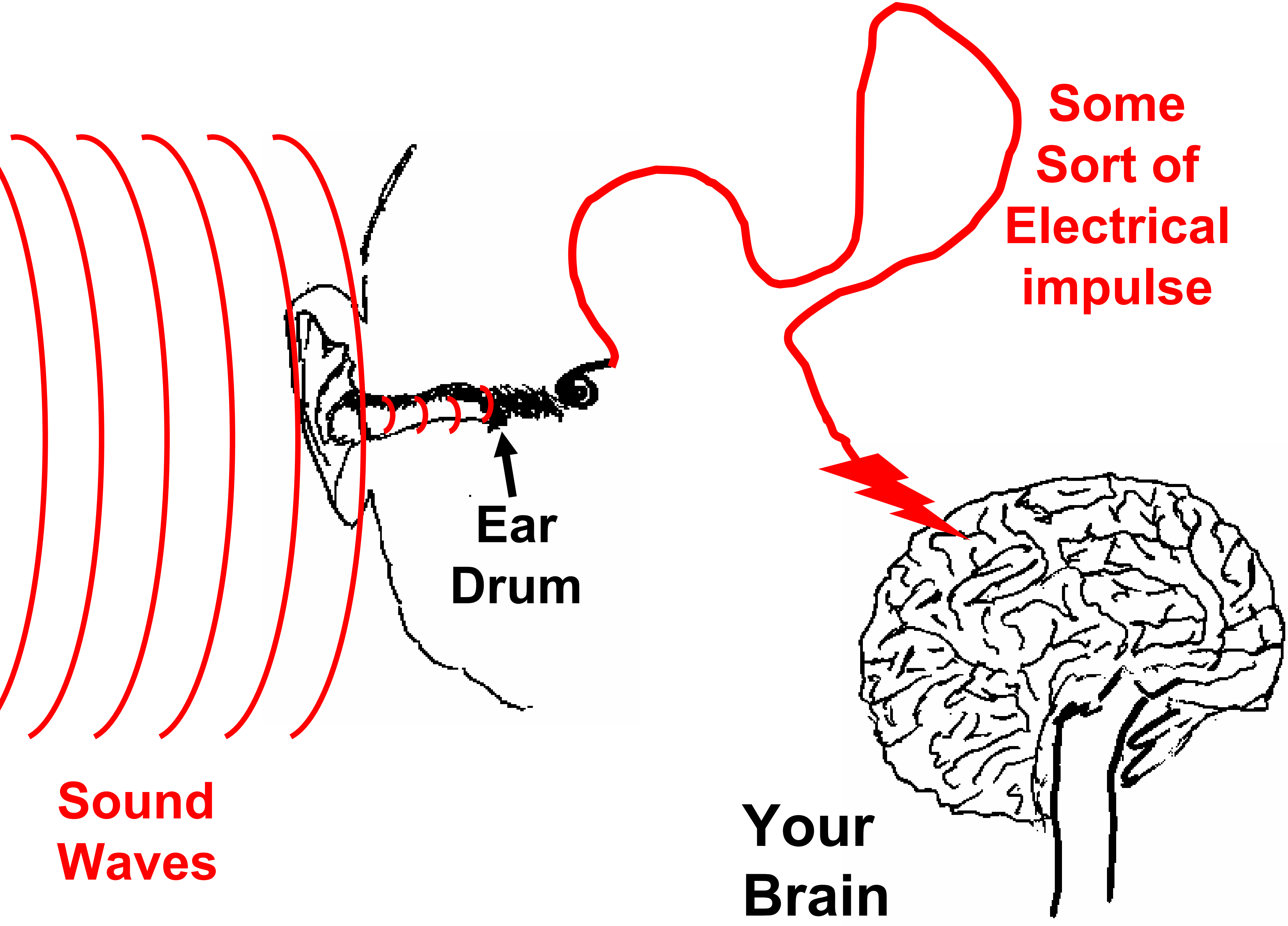
How far away are stars?   How big are stars?



Both objects have an <u>angular diameter</u> of 3°


Run 94 Col 3 Field 387

They have different apparent brightness
They have different colors
They move
They change in brightness


Some
Sort of
Electrical
impulse
Ear
Drum
Sound
Waves
Your
Brain

Loudness: Intensity: energy per second in ear

$I_{THRESHOLD}$ = energy per second in ear at threshold of hearing

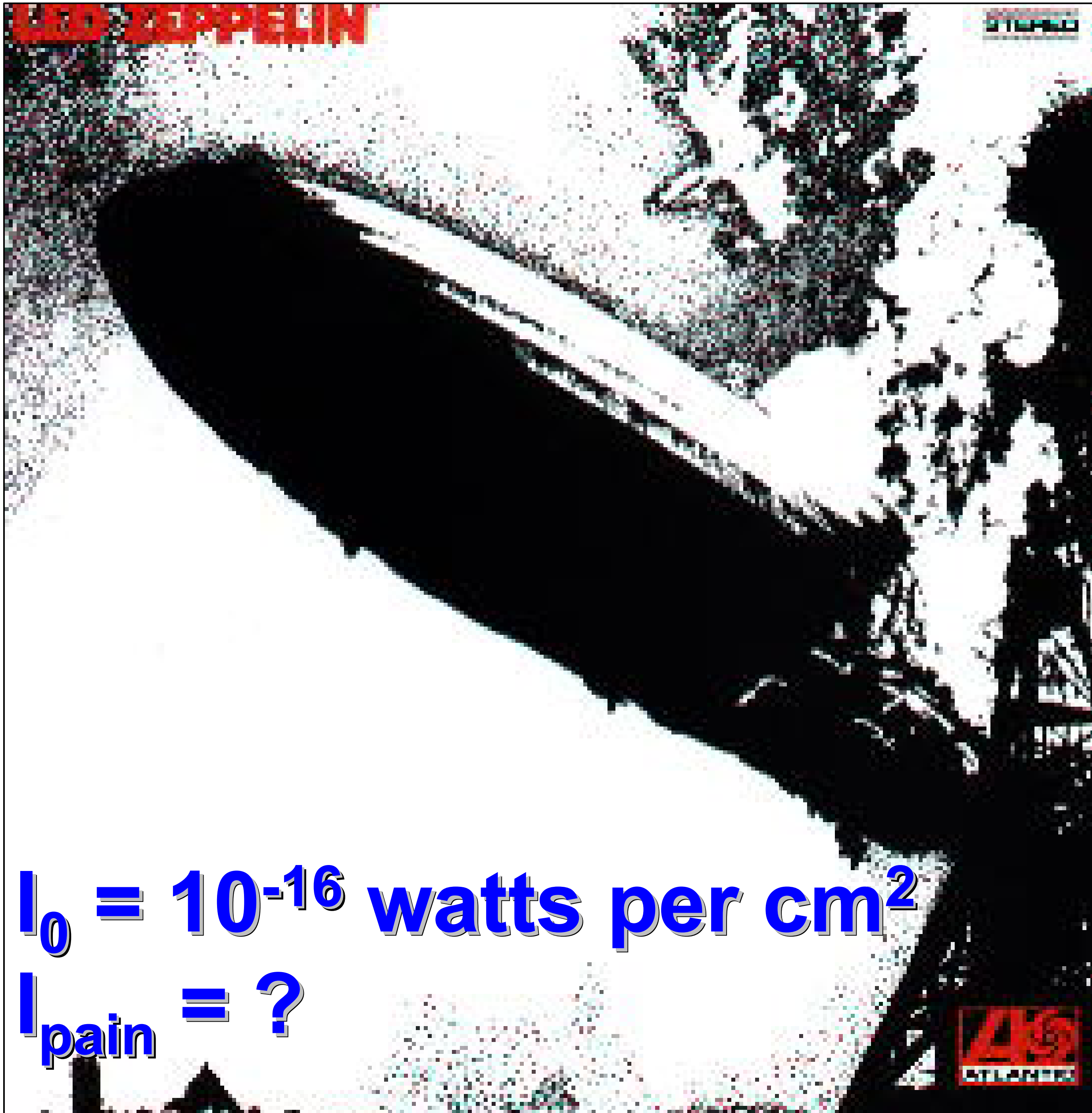
I_{PAIN} = energy per second in ear at threshold of pain

$I_{PAIN} / I_{THRESHOLD}$ = ?

Hearing threshold

$I_0 = 10^{-16}$ watts per cm^2

LED ZEPPELIN



I_0 = 10^{-16} watts per cm^2

I_{pain} = ?

Loudness: Intensity: energy per second in ear

$I_{THRESHOLD}$ = energy per second in ear at threshold of hearing

I_{PAIN} = energy per second in ear at threshold of pain

$I_{PAIN} / I_{THRESHOLD} = 10^{12}$!!!

1 – 100 (10^2)

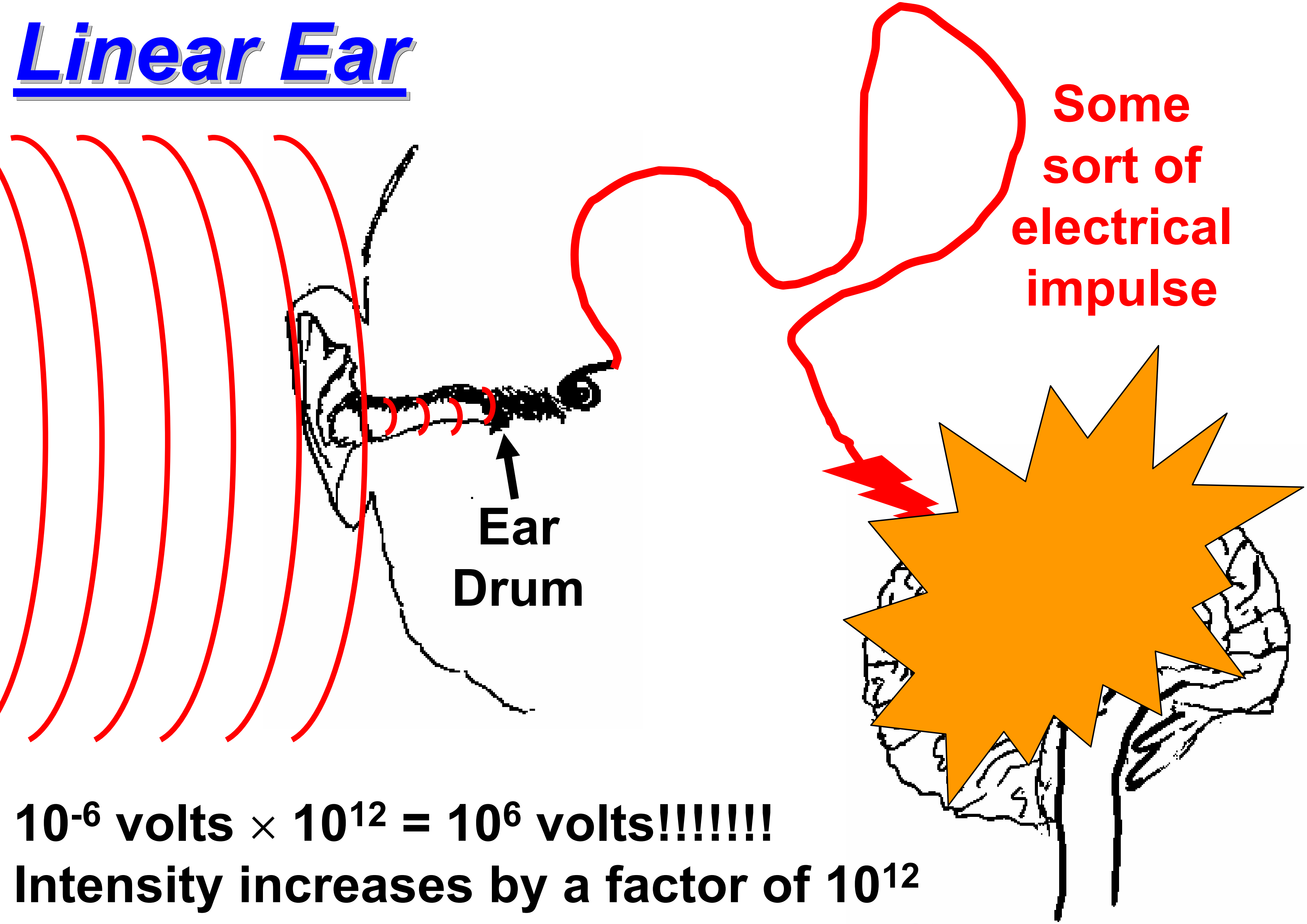
100 – 1,000 (10^3)

1,000 – 1,000,000 (10^6)

1,000,000 – 1,000,000,000 (10^9)

1,000,000,000 – 1,000,000,000,000 (10^{12})

Linear Ear



10^{-6} **volts** $\times$ 10^{12} = 10^{6} **volts!!!!!!!**

Intensity increases by a factor of 10^{12}

$\rightarrow$ **electrical impulse increases by** 10^{12}

Intensity: energy per time per area

$$I = \frac{\text{Energy}}{\text{Time Area}}$$

I_0 = threshold of hearing

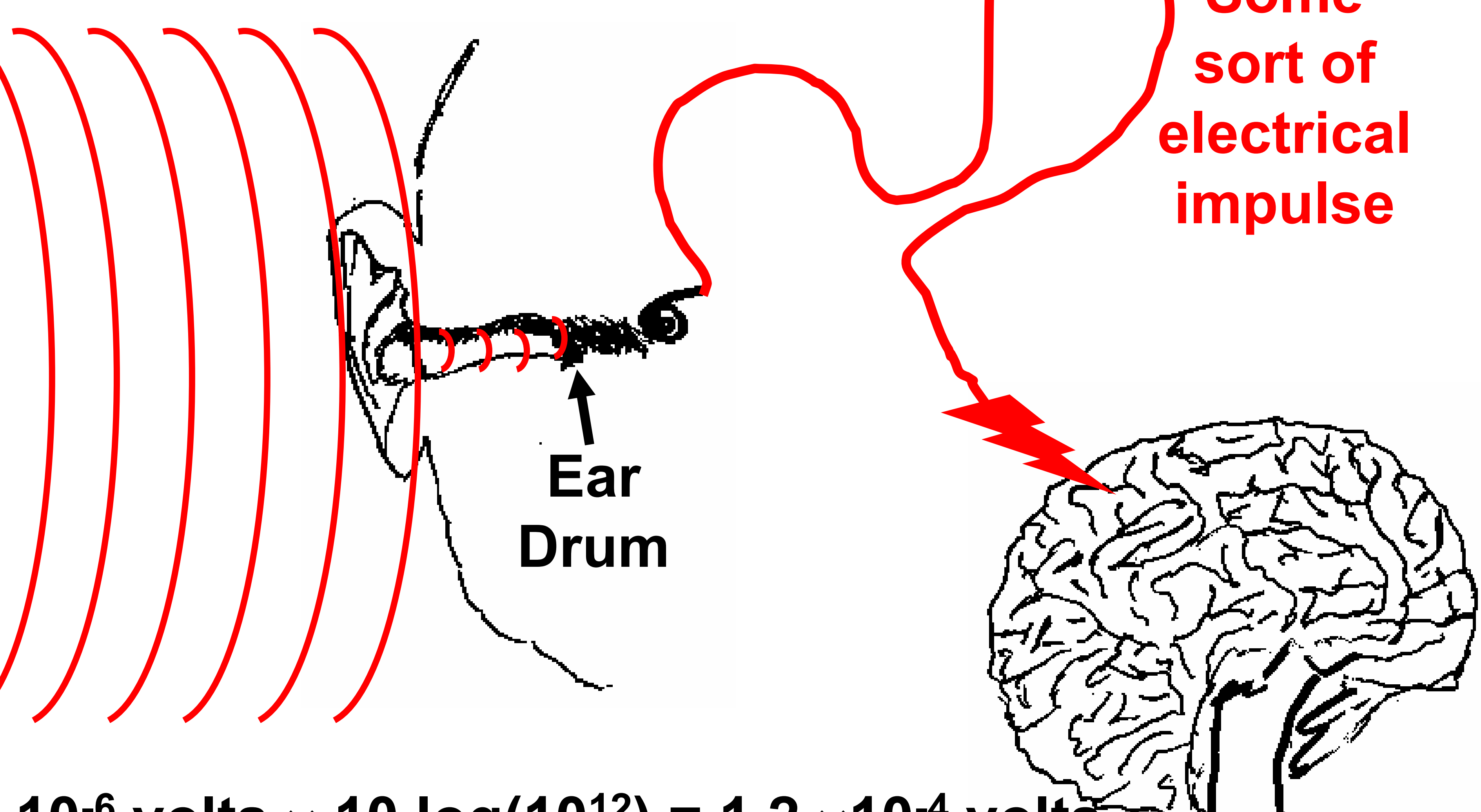
$$dB = 10 \log (I/ I_0)$$

$$I/ I_0 = 10^{12}$$

$$\log (10^{12}) = 12$$

$$dB = 10 \times 12 = 120$$

Logarithmic Ear



10^{-6} volts $\times$ 10 log(10^{12}) = 1.2 $\times 10^{-4}$ volts

Electrical impulse (output) only increases as the <u>logarithm</u> of the input

I_0 is intensity at threshold of hearing

I/I_0	$\log(I/I_0)$	$dB = 10 \log(I/I_0)$
10^{-2}	-2	-20
1	0	0
10^{2}	2	20
10^{6}	6	60
10^{12}	12	120
10^{20}	20	200

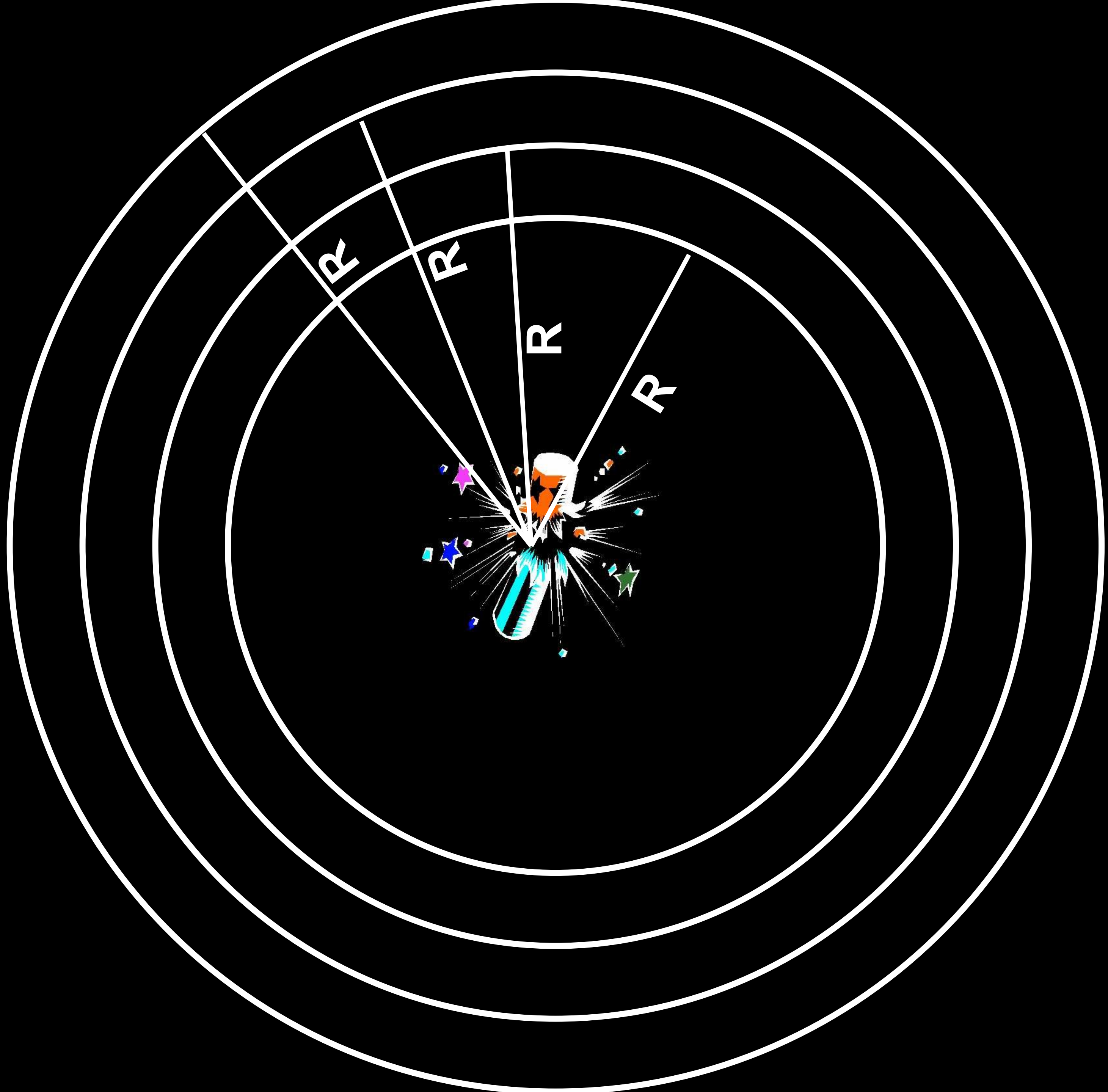
Intensity: energy per time per area

$$I = \frac{\text{Energy}}{\text{Time Area}} = \frac{\text{Power}}{\text{Area}}$$

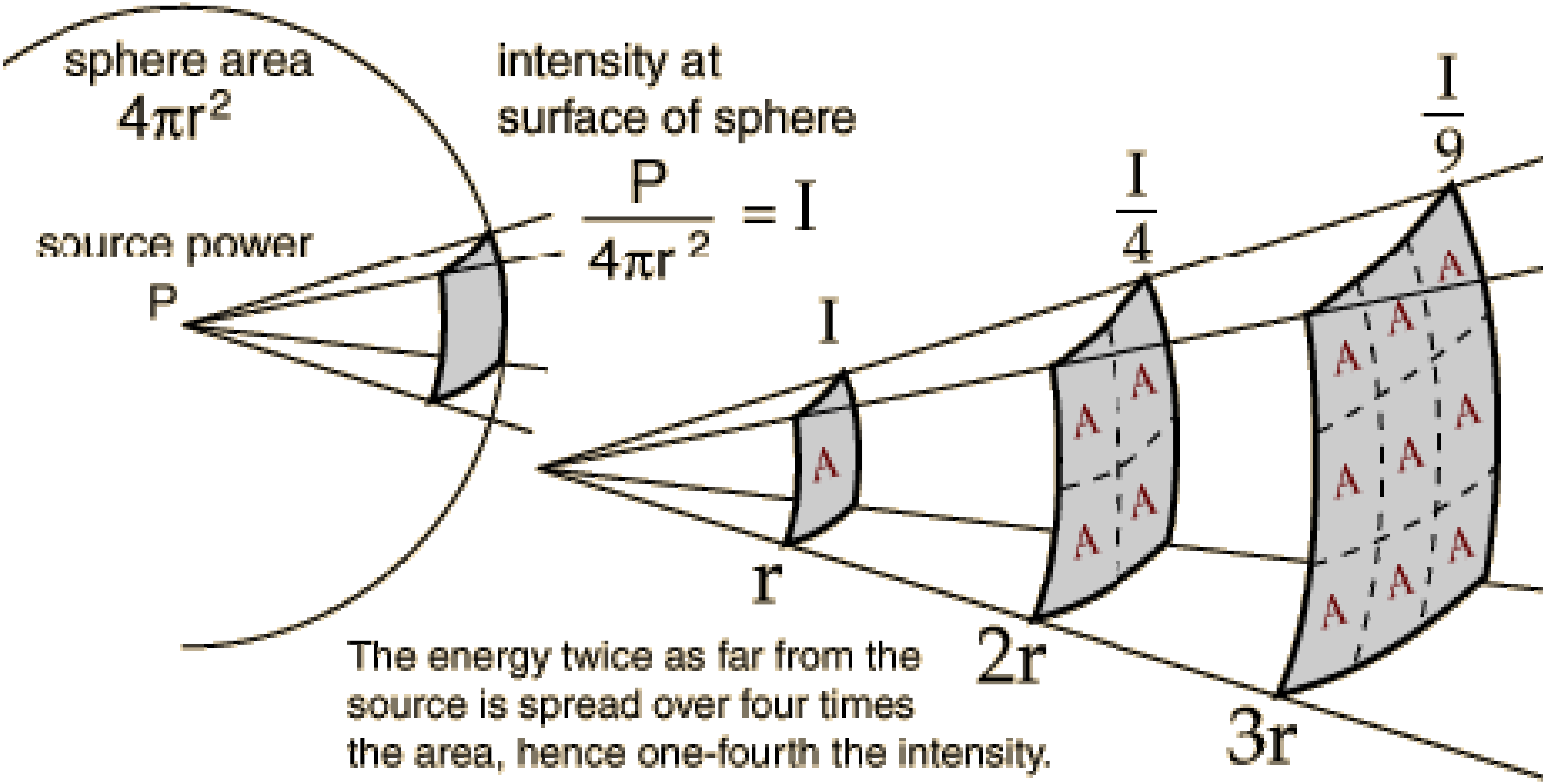
$\frac{\text{Energy}}{\text{Time}}$ (Power) measured in watts

Area measured in cm^2

Intensity in watts per cm^2


R
R
R
R

Inverse-square law



Intensity: energy per time per area

$$I = \frac{\text{Energy}}{\text{Time Area}} = \frac{\text{Power}}{\text{Area}}$$

<u>**Power**</u> property of source

<u>**Intensity**</u> depends on power and distance between source and detector

$$\text{Intensity} = \frac{\text{power}}{4\pi R^2}$$

Let there be light

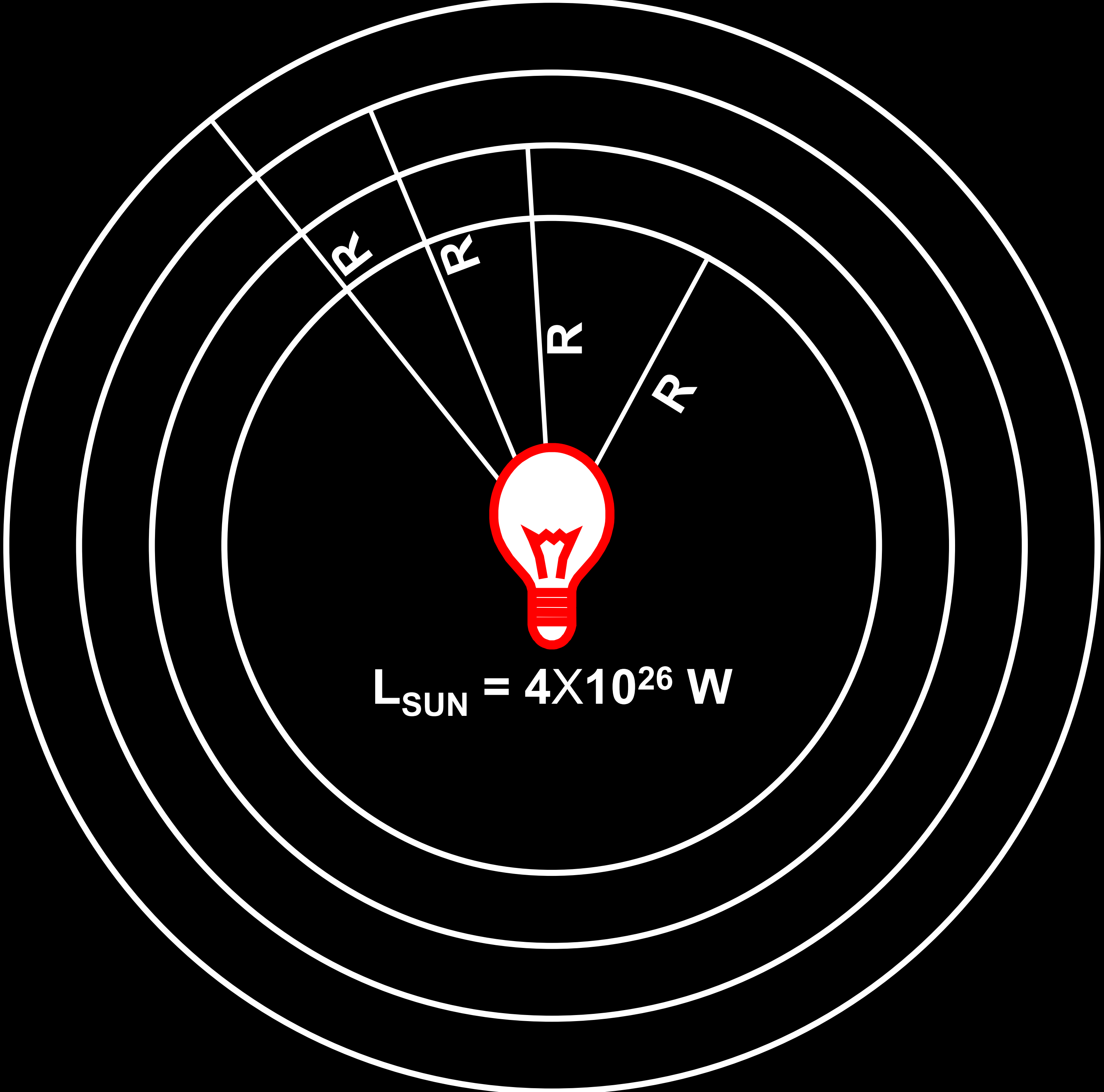
For light!!!

$$I = \frac{\text{Energy}}{\text{Time Area}}$$

$\frac{\text{Energy}}{\text{Time}}$ (Luminosity) measured in watts

Area measured in cm^2

Intensity in watts per cm^2


R
R
R
R
$L_{SUN} = 4 \times 10^{26}$ W

For light!!!

$$I = \frac{\text{luminosity}}{\text{cm}^2}$$

Luminosity property of source

Intensity depends on power and distance between source and detector

$$\text{Intensity} = \frac{\text{luminosity}}{4\pi R^2}$$

Logarithmic Eye

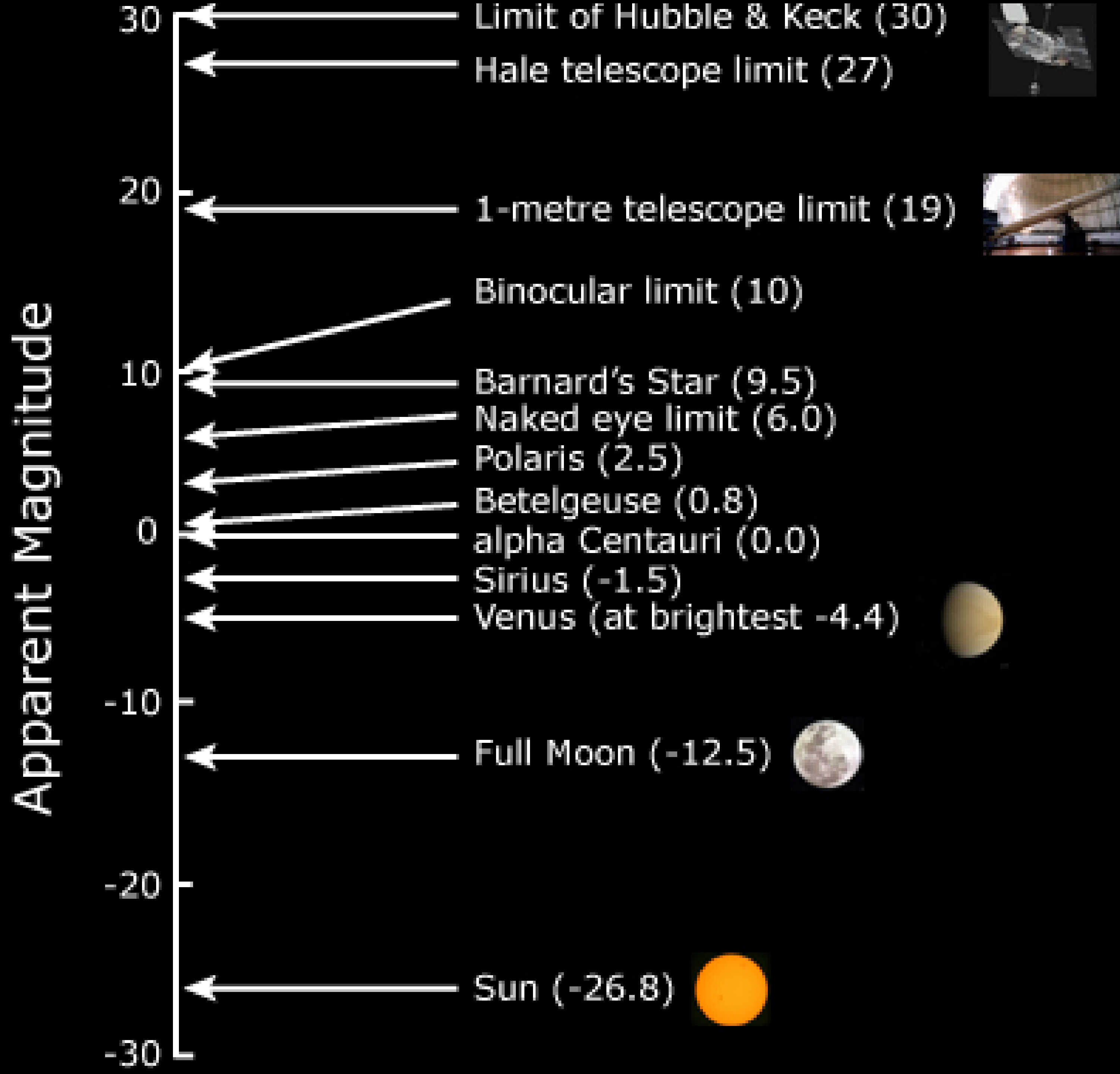
Eyes, like ears, are logarithmetic detectors.

LET THERE BE LIGHT!

Greeks classified stars into 6 classes, or magnitudes

Brightest stars were 1st magnitude
Dimmest stars were 6th magnitude

Intensity of brightest stars = 100 X dimmest.


Apparent Magnitude
30
20
10
0
-10
-20
-30
Limit of Hubble & Keck (30)
Hale telescope limit (27)
1-metre telescope limit (19)
Binocular limit (10)
Barnard's Star (9.5)
Naked eye limit (6.0)
Polaris (2.5)
Betelgeuse (0.8)
alpha Centauri (0.0)
Sirius (-1.5)
Venus (at brightest -4.4)
Full Moon (-12.5)
Sun (-26.8)

The luminosity of nearby stars?

Measure: intensity of light, I

parallax → distance

$$I = \frac{L}{4\pi R^2}$$

$$\frac{D}{\text{pc}} = \frac{\text{seconds}}{\text{parallax}}$$

$$I = \frac{L}{4\pi R^2}$$

star	parallax (")	distance (pc)	apparent magnitude	luminosity (solar)
α Centauri	0.75	1.3	0	1.5
Barnard's star	0.5	2.0	9.5	0.0005
Sirius	0.4	2.5	-1.5	25
Altair	0.2	5.0	0.8	10
Canopus	0.003	330	- 0.7	200,000
Arcturus	0.1	10	0	90
Betelgeuse	0.01	100	0.5	14,000

Our Sun ain't the brightest bulb in the box!

$$\text{Intensity} = \frac{\text{Luminosity}}{4\pi R^2}$$

$$L_{SIRIUS} = 25 \times L_{SUN}$$

For stars we know distance to via parallax:

Measure Distance (R)
Measure Intensity

$\longrightarrow$ Know Luminosity

They have different apparent brightness

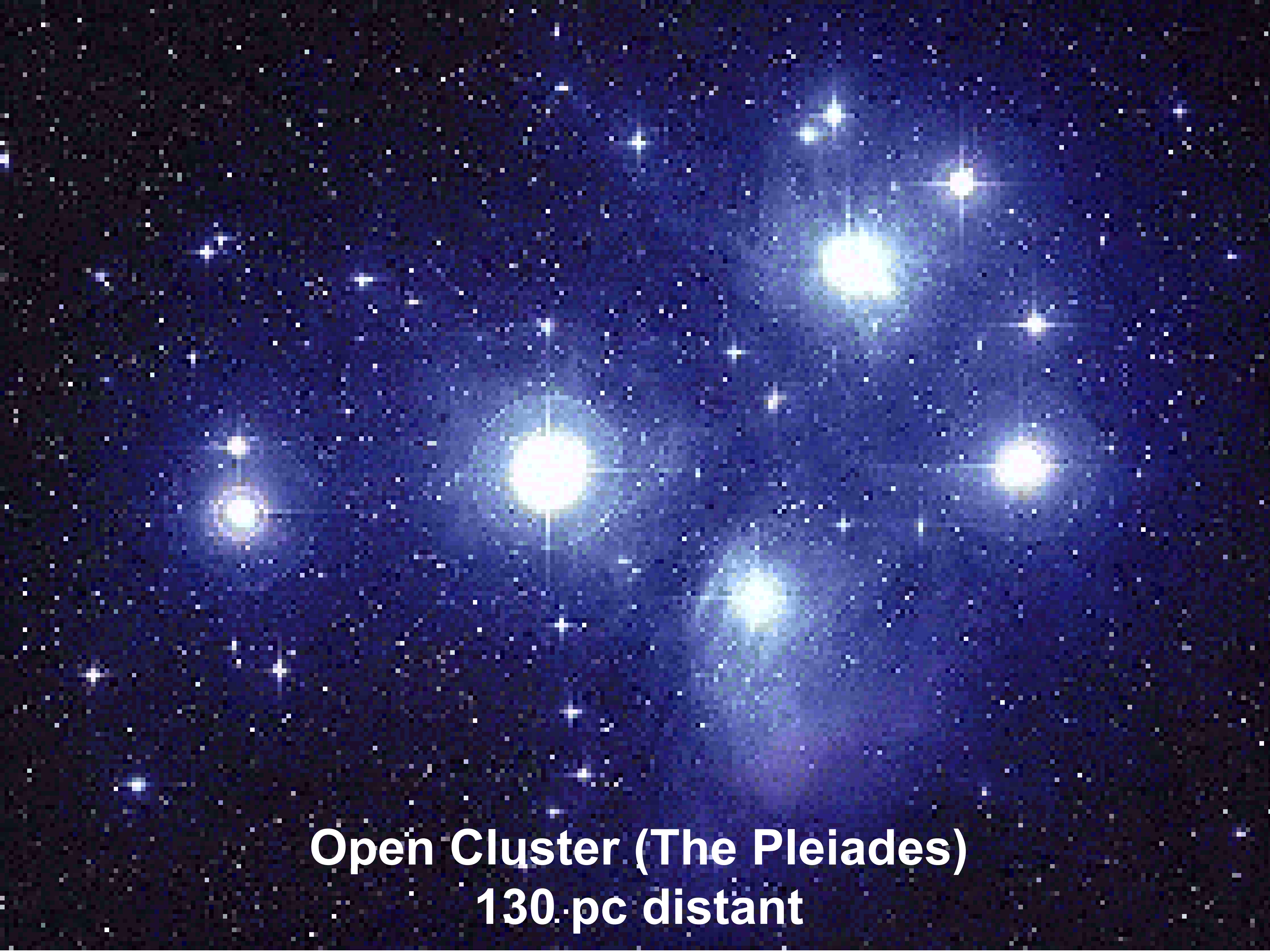
They have different colors

They move

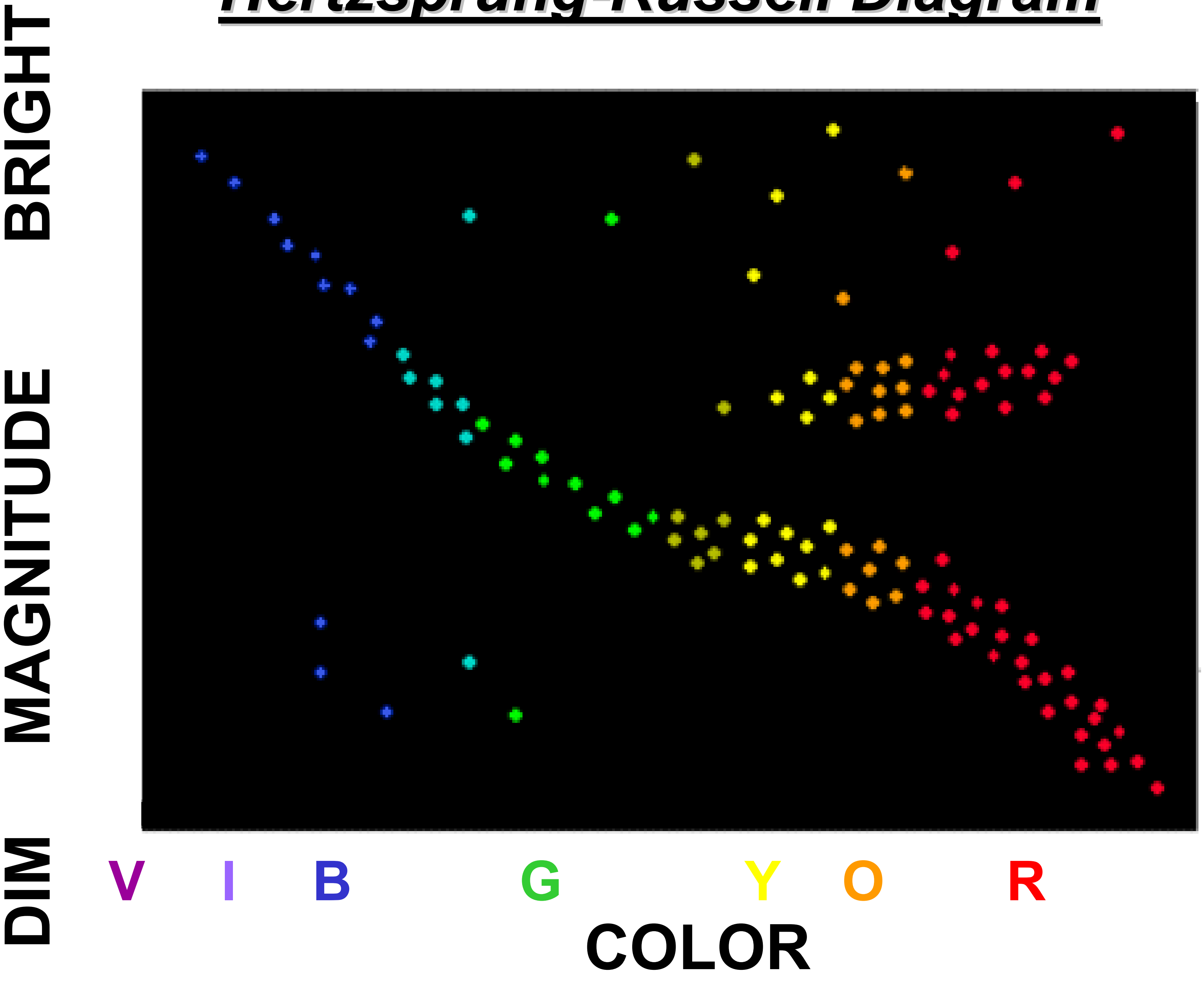
They change in brightness

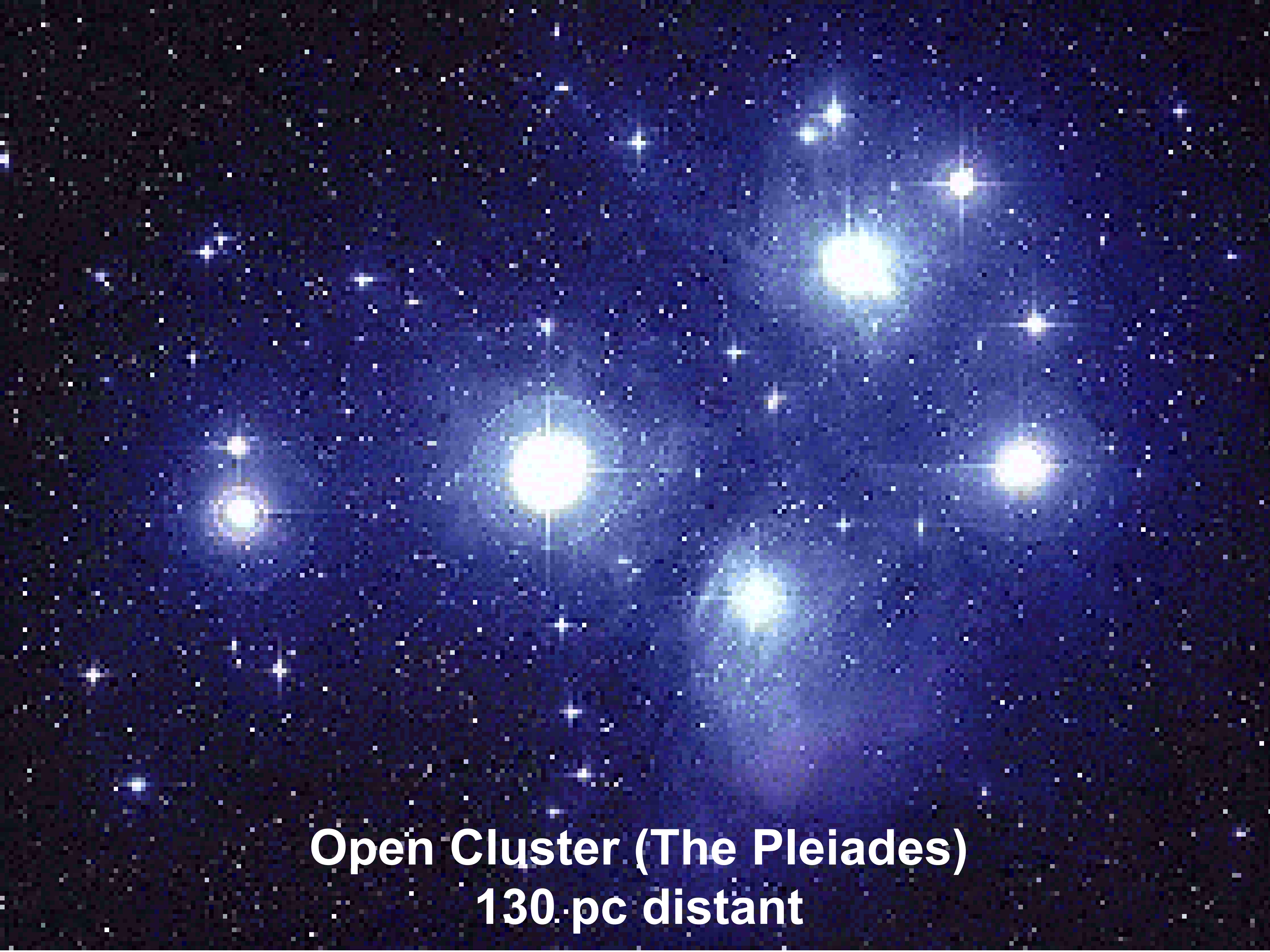
COLORS OF THE RAINBOW:

R O Y G B I V

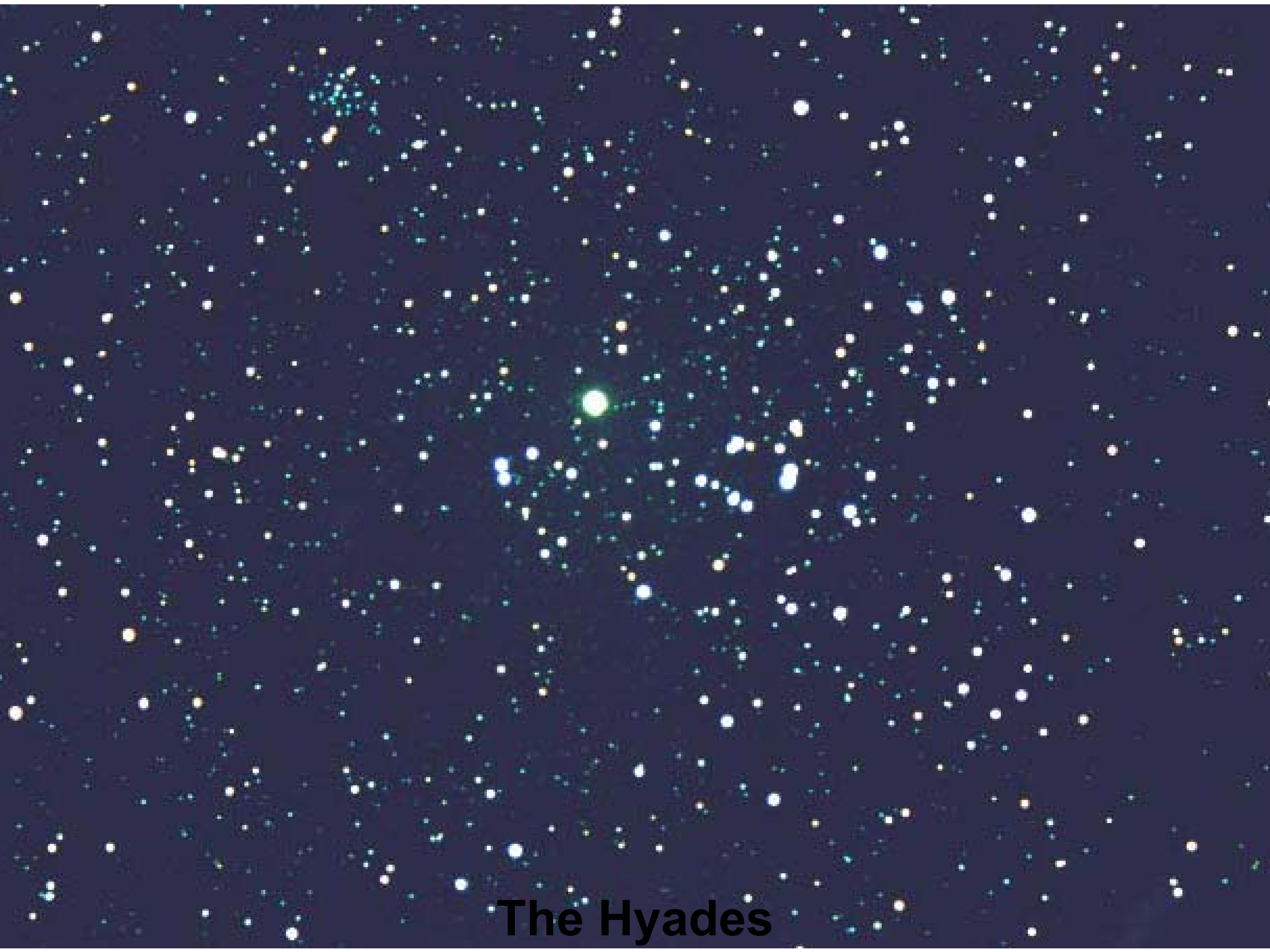

Open Cluster (The Pleiades)
130 pc distant

Hertzsprung-Russell Diagram

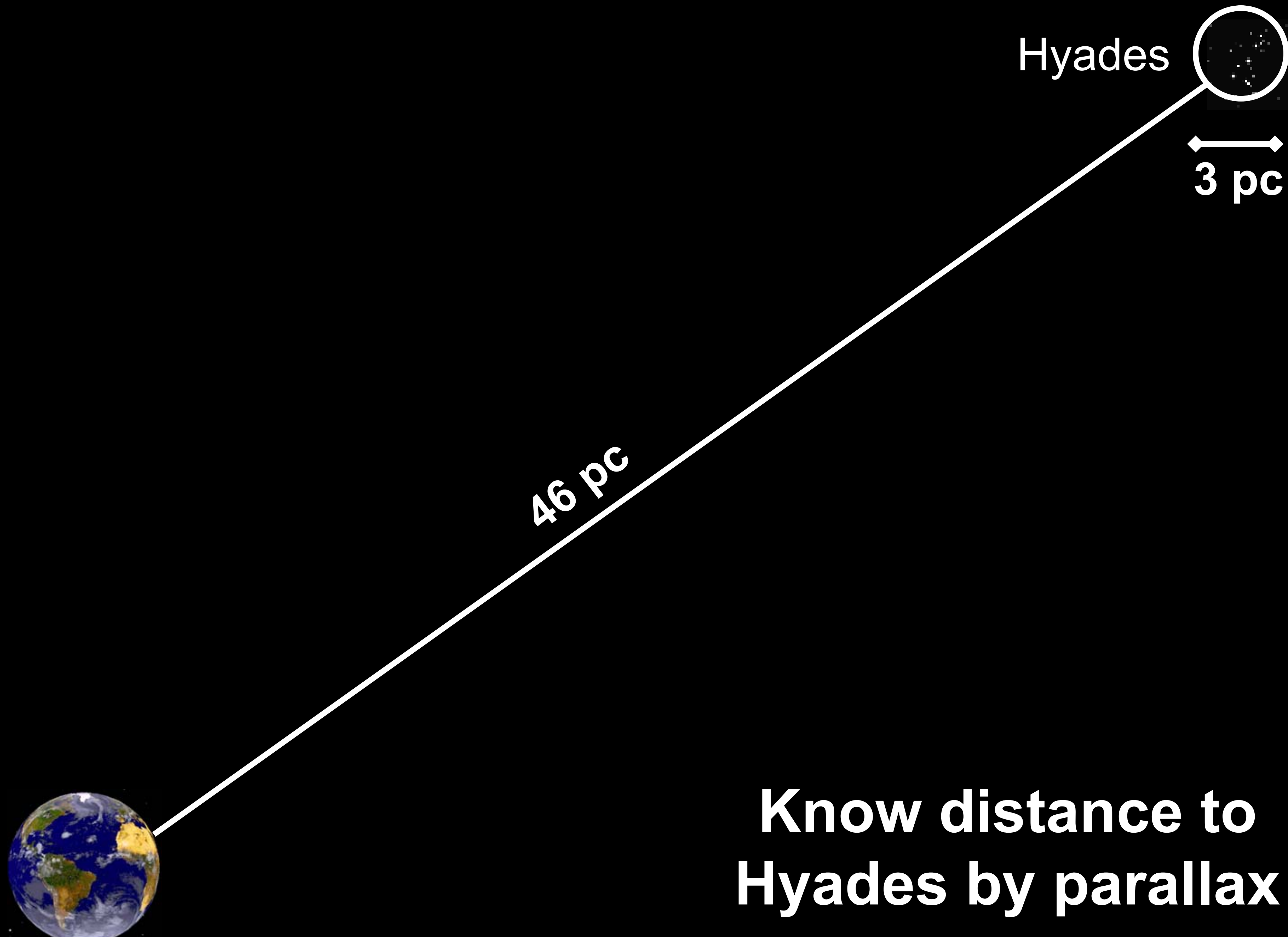



Open Cluster (The Pleiades)
130 pc distant


M 45 (Pléiades)
Hyades
© Denis Bergeron 1980



The Hyades

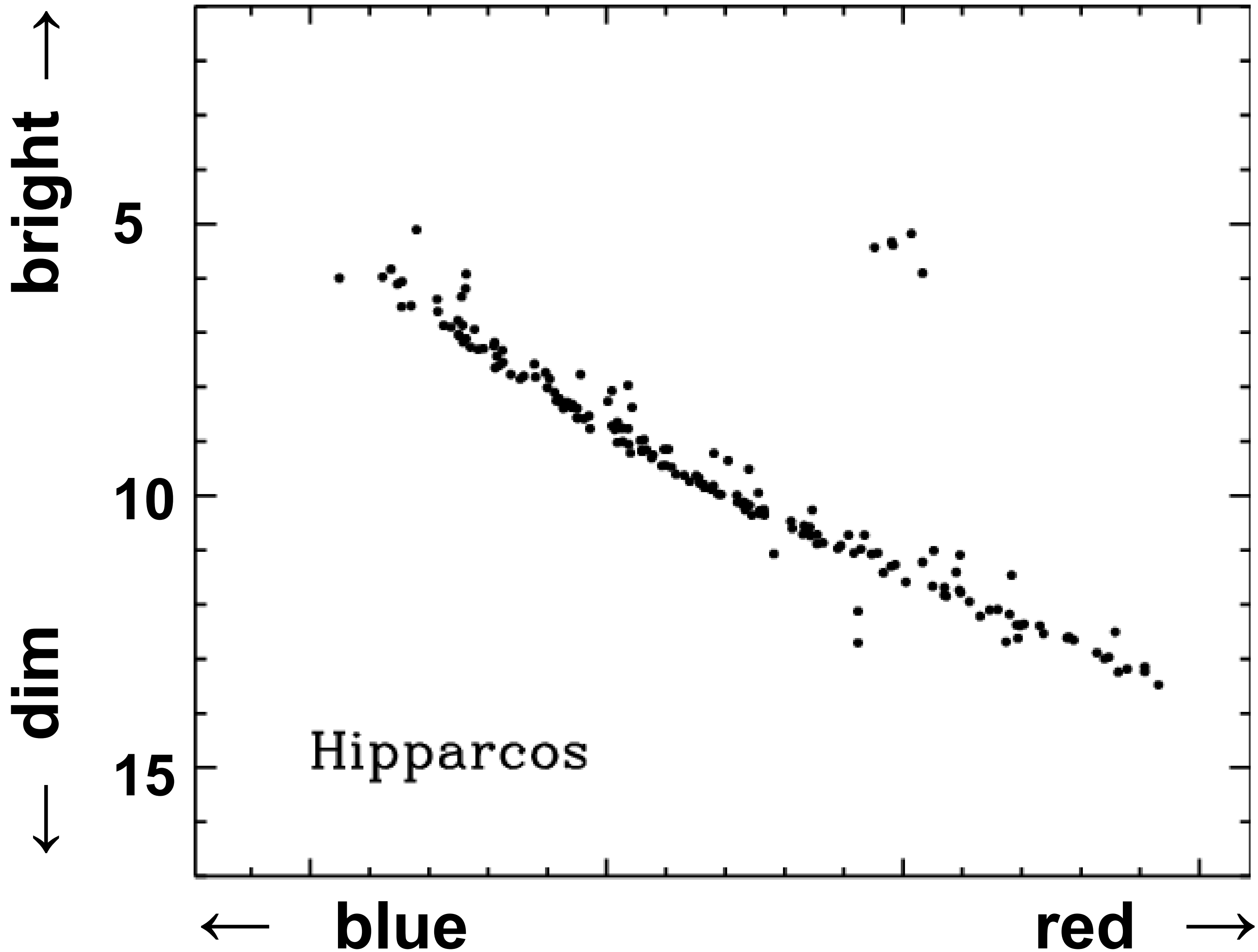


Know distance to Hyades by parallax

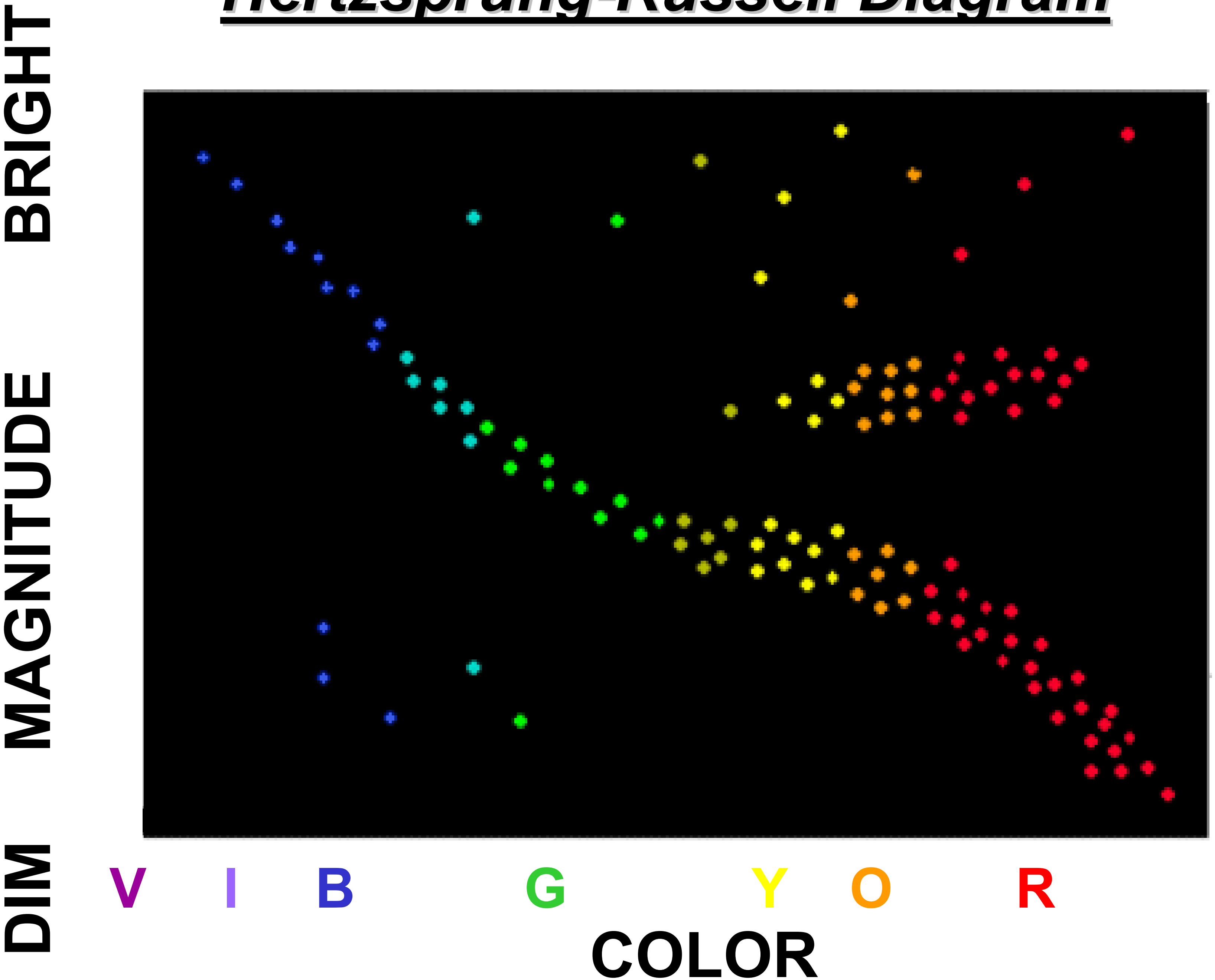
Ejnar Hertzsprung (1873-1967)

Henry Russell (1877-1957)

Hyades HR diagram

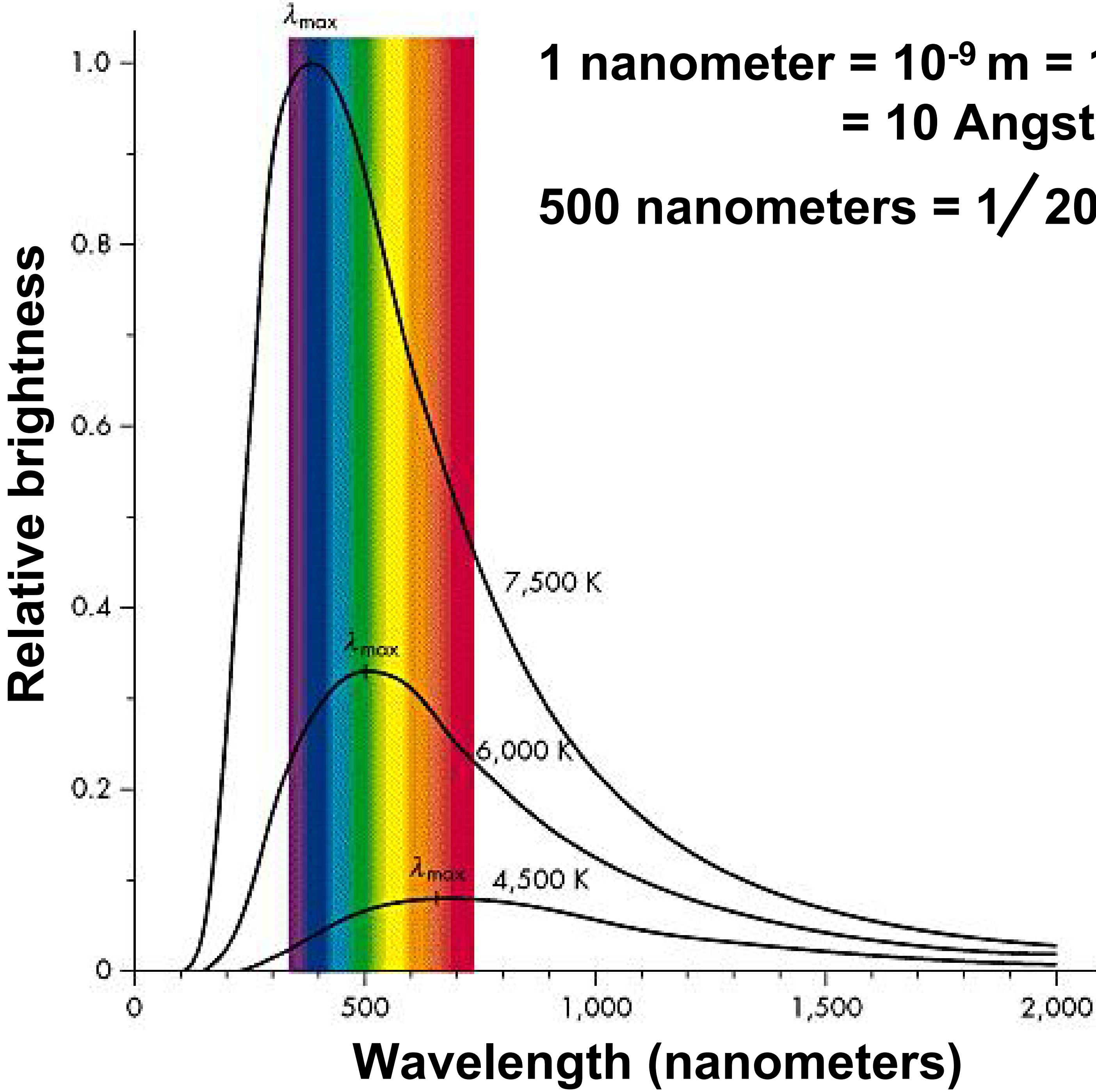


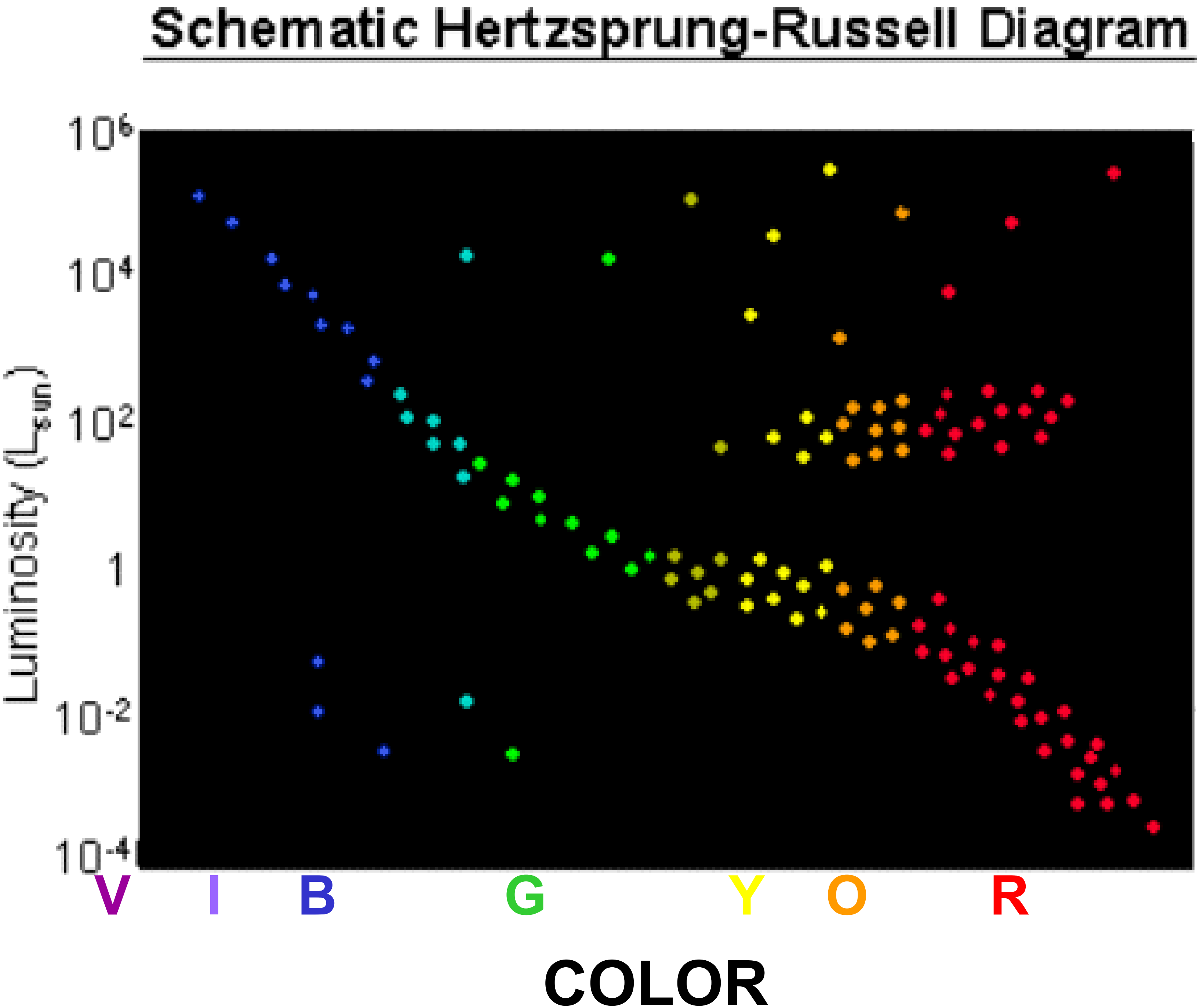
Hertzsprung-Russell Diagram



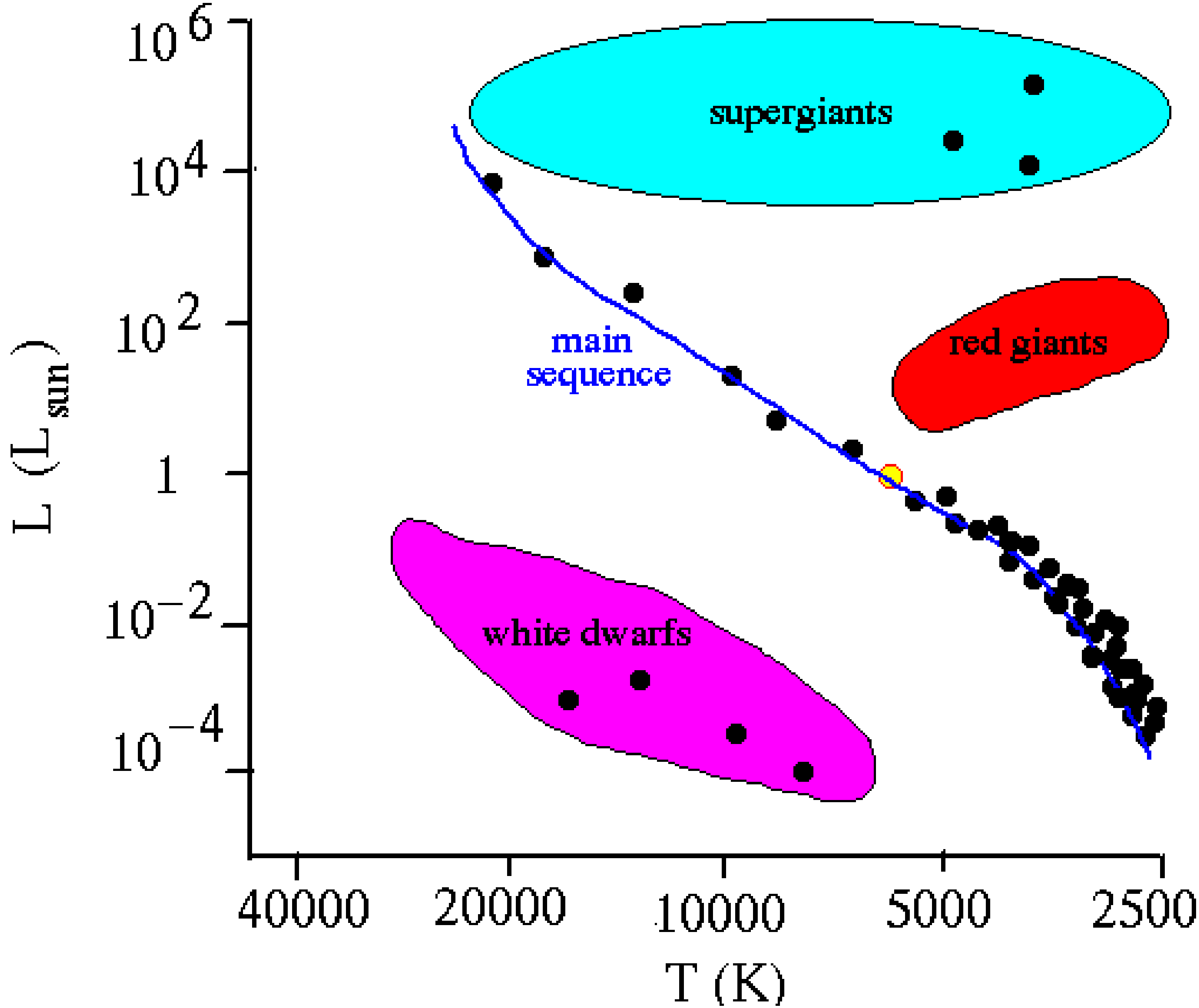
1 nanometer = 10^{-9} m = 10^{-7} cm
= 10 Angstroms

500 nanometers = 1/20,000 cm

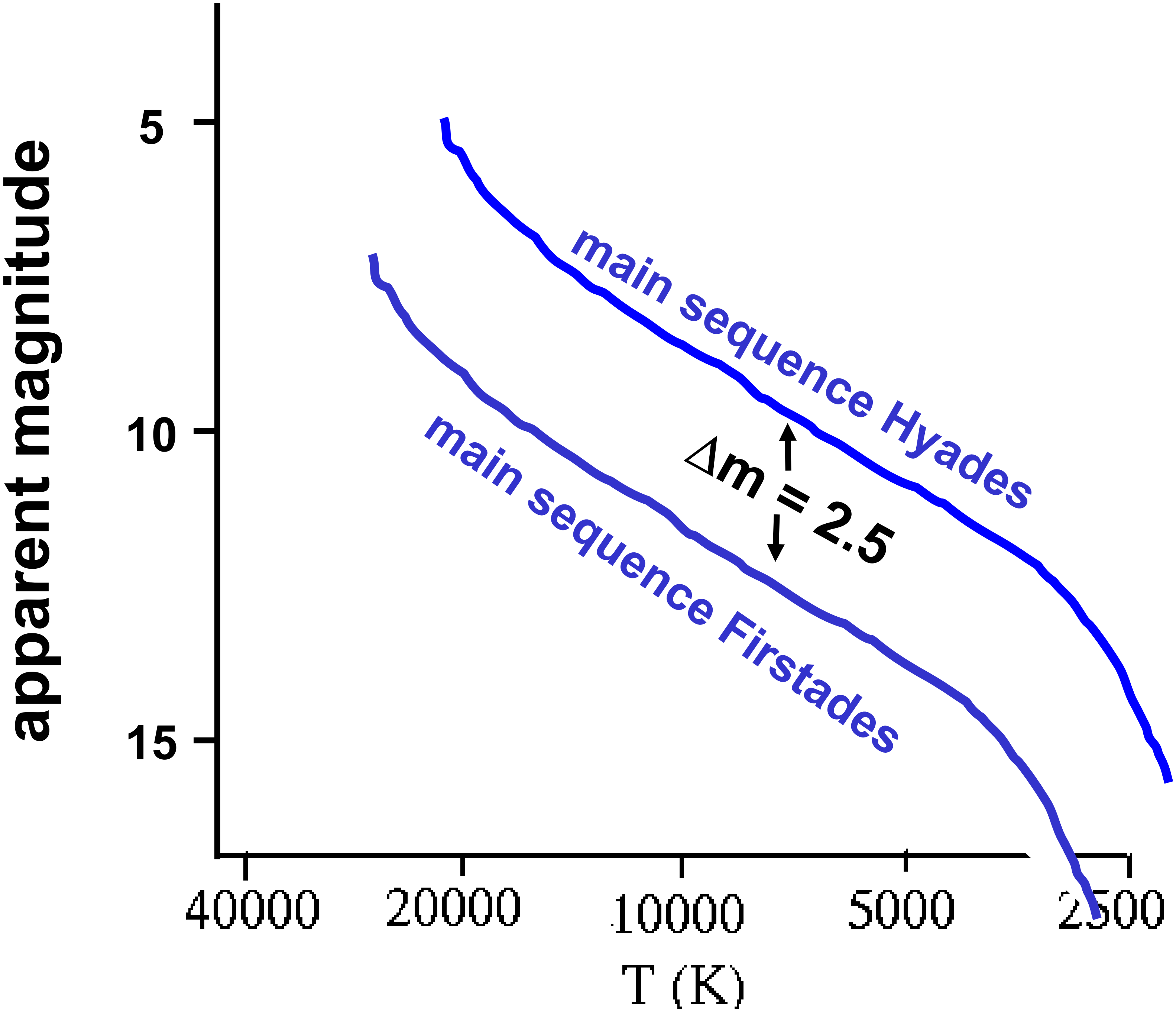




Schematic Hertzsprung-Russell Diagram
Luminosity (L_{sun})
10^6
10^4
10^2
1
10^{-2}
10^{-4}
V
I
B
G
Y
O
R
COLOR



10^6
10^4
10^2
1
10^{-2}
10^{-4}
L (L_{sun})
supergiants
main sequence
red giants
white dwarfs
40000
20000
10000
5000
2500
T (K)



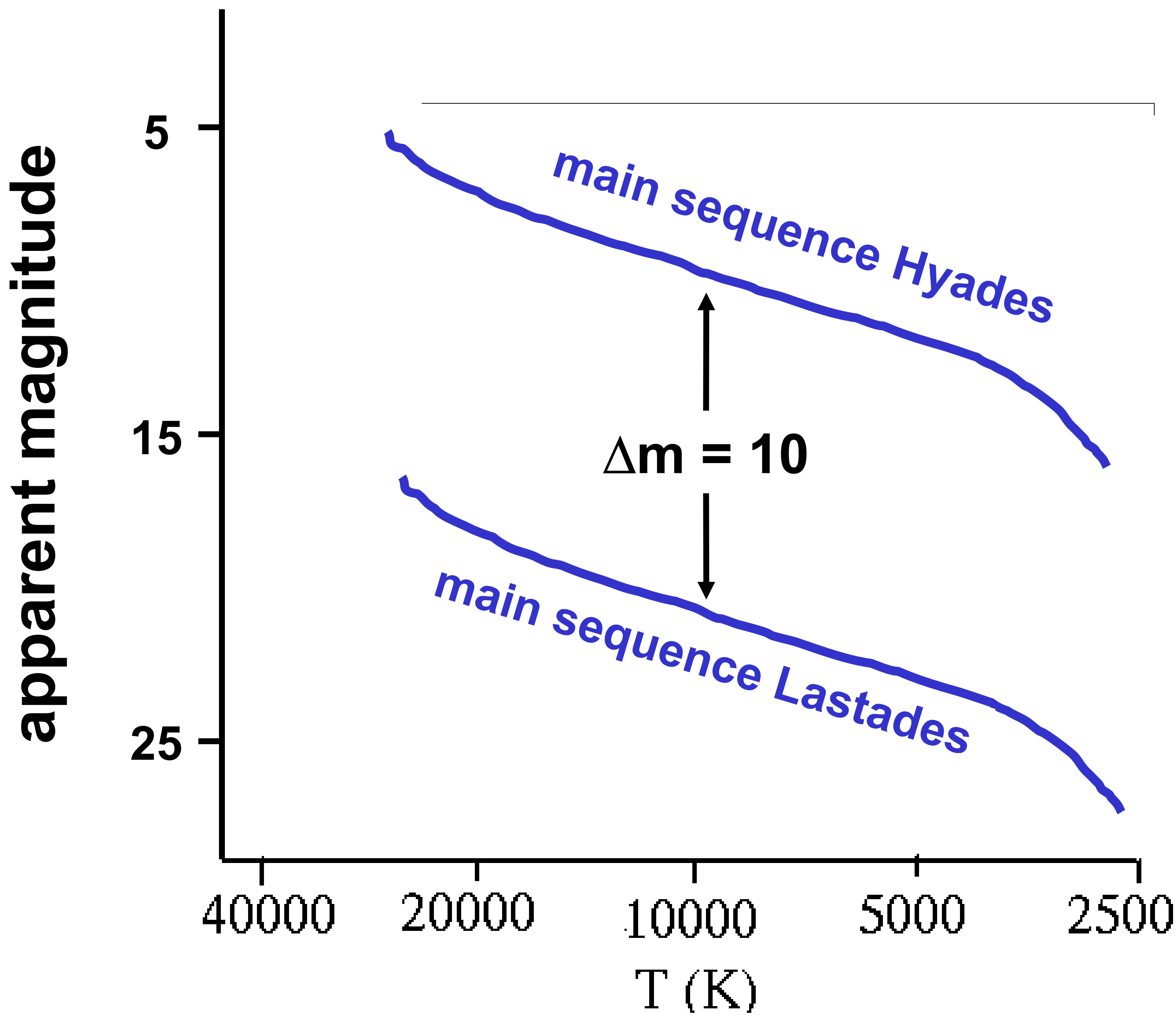
apparent magnitude
5
10
15
40000
20000
10000
5000
2500
T (K)
main sequence Hyades
main sequence Firstades
Δm = 2.5

If Δm between Firstades and Hyades only due to difference in distance, then can express distance to Firstades in terms of distance to Hyades.

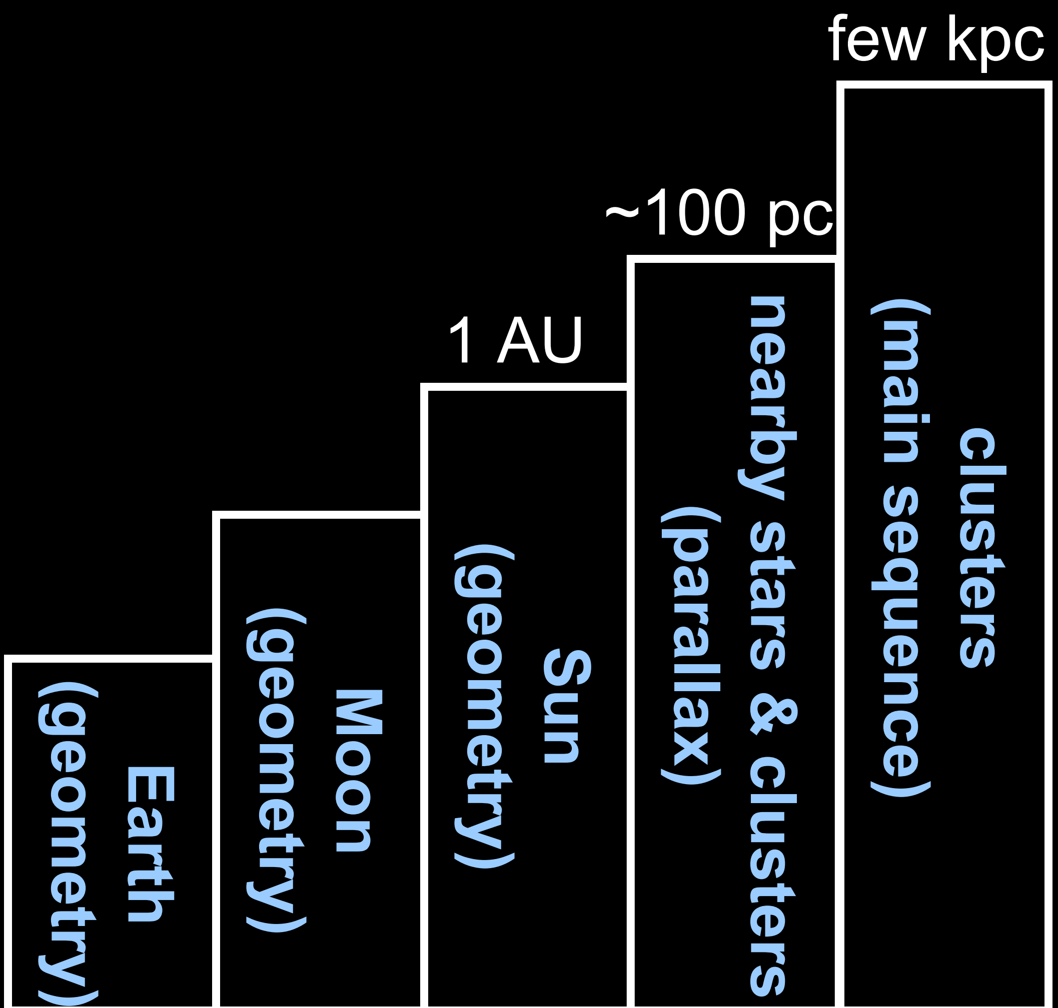
$$\frac{I_H}{I_F} = \frac{R_F^2}{R_H^2} \qquad 10 = \frac{R_F^2}{R_H^2} \qquad 3 = \frac{R_F}{R_H}$$

Distances to other clusters

- **Construct H-R diagram for cluster**
- **Measure Δm compared to HR diagram for Hyades**
- **Compute distance in terms of distance to Hyades**
- **How far can you go?**
- **Say most distant open observable cluster is Lastades**


apparent magnitude
5
15
25
main sequence Hyades
Δm = 10
main sequence Lastades
40000
20000
10000
5000
2500
T (K)

The Cosmological Distance Ladder

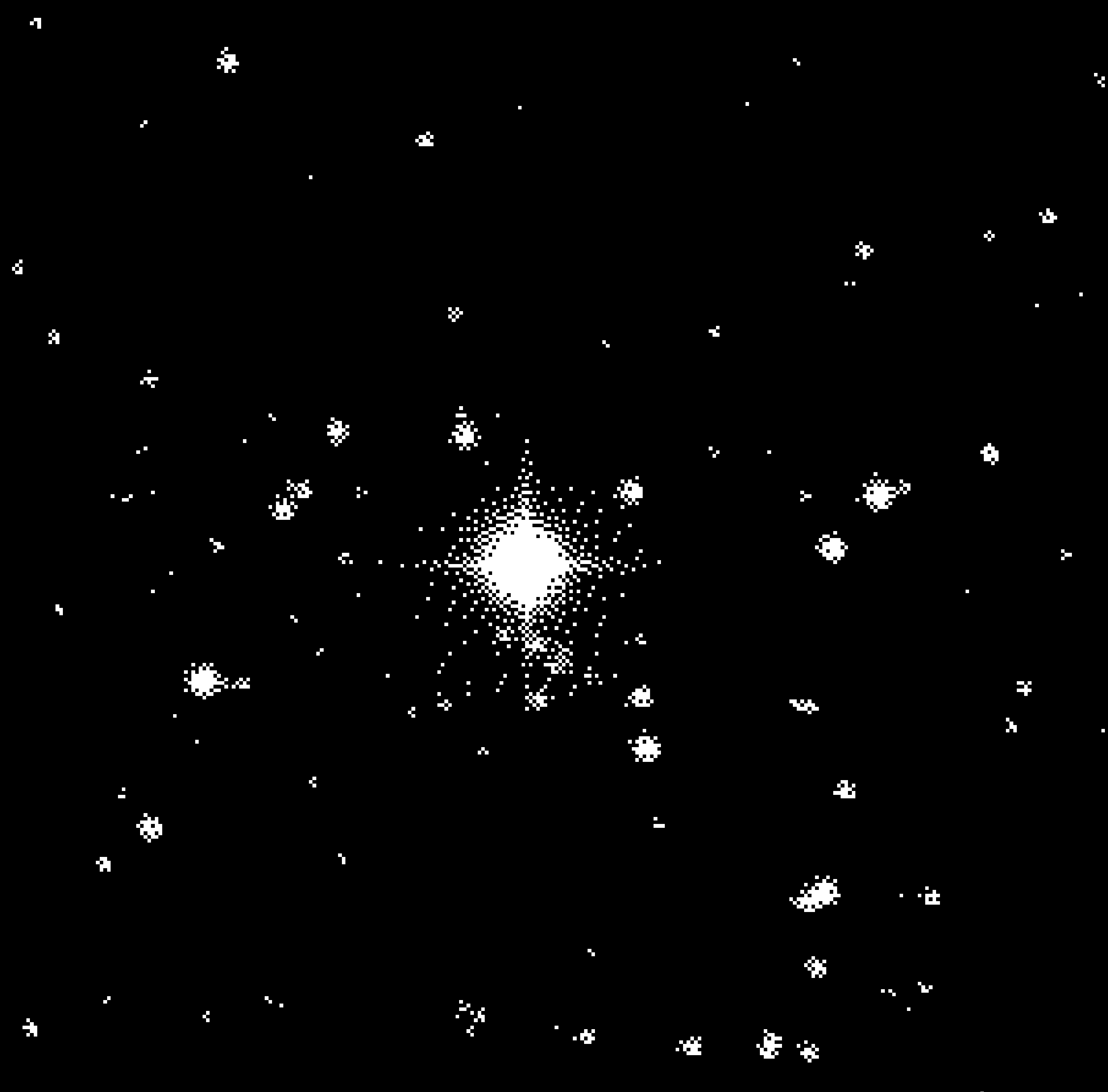


- Main sequence stars are not extremely bright… we need brighter “standard candle”

$$\text{Intensity} = \frac{\text{Luminosity}}{4\pi R^2}$$

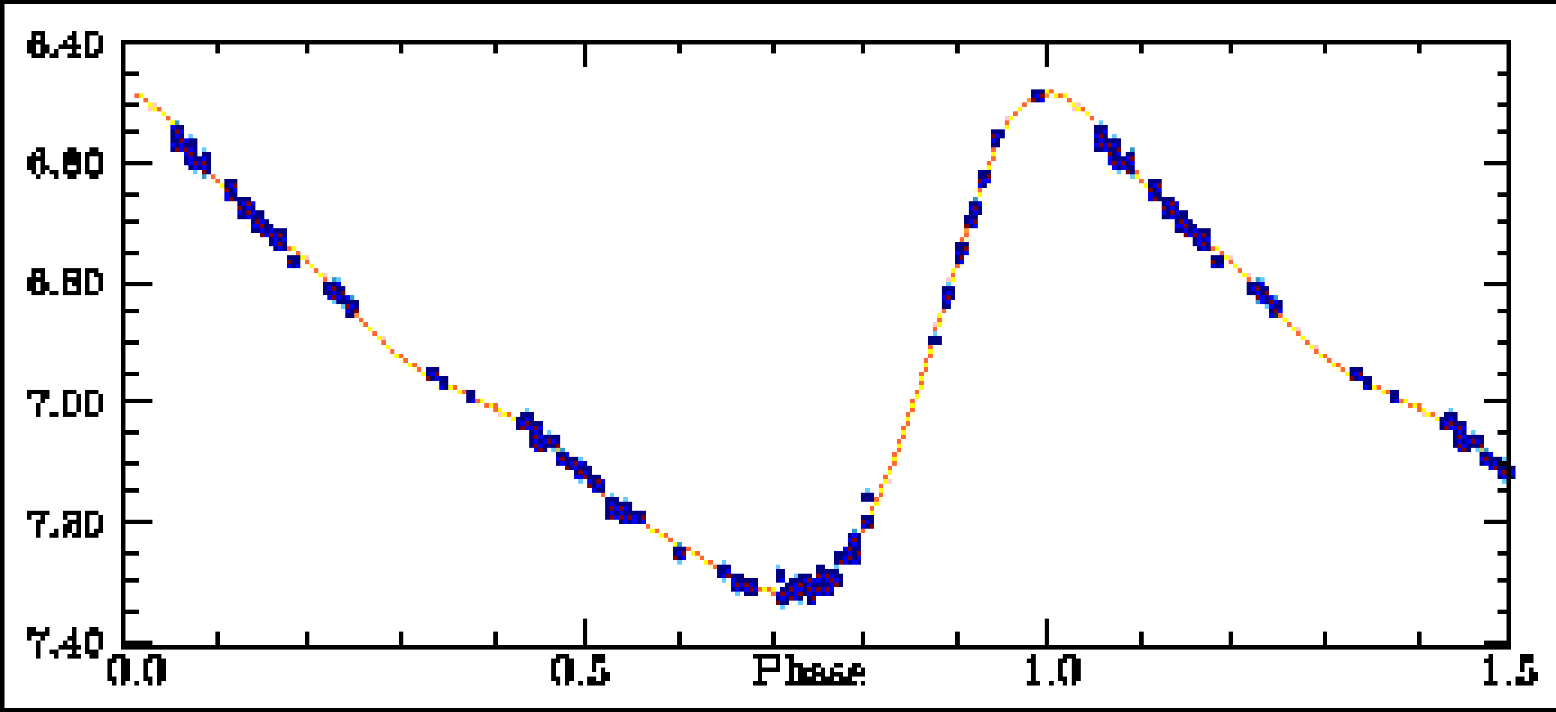
They move
They have different apparent brightness
They have different colors
They change in brightness

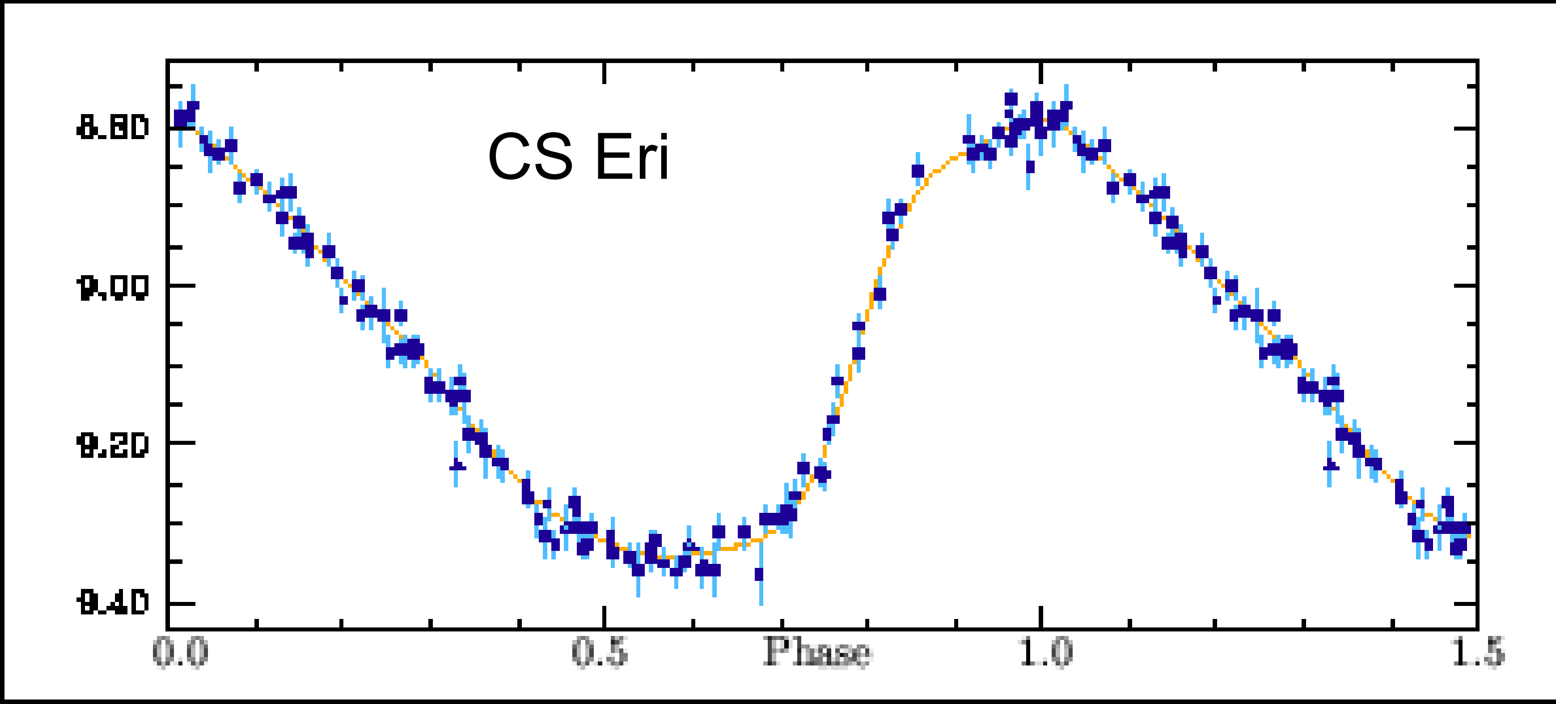
RR Lyrae Stars



RR Lyrae Stars

- Class named after a particular star: RR Lyrae
- Compared to the sun
  - half the mass
  - older than sun
  - hotter
  - expended hydrogen … burning helium to carbon
  - pulsates
- Changes brightness with regular period of days
- Luminosity determined by size & temperature
  - for same temperature: larger $\rightarrow$ more luminous
  - for same size: hotter $\rightarrow$ more luminous
- Shrink $\rightarrow$ compressional heating $\rightarrow$ more luminous


6.40
6.60
6.80
7.00
7.20
7.40
0.0
0.5
Phase
1.0
1.5


CS Eri
8.80
9.00
9.20
9.40
0.0
0.5
Phase
1.0
1.5

- Main sequence stars are not extremely bright… we need brighter "standard candle"

$$\text{Intensity} = \frac{\text{Luminosity}}{4\pi R^2}$$

- RR Lyrae stars found in distant clusters we know the distance to via H-R fitting.

- RR Lyrae stars are identified because their light output changes regularly on a time scale of half to one day.

- They are brighter than the sun by about a factor of 100 and are standard candles. Can see farther away and use as standard candle.

The Cosmological Distance Ladder